

8. MATHEMATICS

Results of Secondary 3 Mathematics in TSA 2009

The territory-wide percentage of S.3 students achieving Mathematics Basic Competency in TSA 2009 was 80.0%. In 2008 the percentage was 79.8%.

Secondary 3 Assessment Design

The design of assessment tasks for S.3 was based on the documents *Mathematics Curriculum: Basic Competency for Key Stage 3 (Tryout Version)* and *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5), 1999*. The tasks covered the three dimensions of the mathematics curriculum, namely **Number and Algebra**, **Measures, Shape and Space**, and **Data Handling**. They focused on the Foundation Part of the S1 – 3 syllabuses in testing of the relevant concepts, knowledge, skills and applications.

The Assessment consisted of various item types including multiple-choice questions, fill in the blanks, answers-only questions and questions involving working steps. The item types varied according to the contexts of the questions. Some test items consisted of sub-items. Besides finding the correct answers, students were also tested in their ability to present solutions to problems. This included writing out the necessary statements, mathematical expressions and explanations.

The Assessment consisted of 159 test items (221 score points), covering all of the 129 Basic Competency Descriptors. These items were organized into four sub-papers, each 65 minutes in duration and covering all three Dimensions. Some items appeared in more than one sub-paper to act as inter-paper links. Each student was required to attempt one sub-paper only.

The composition of the sub-papers was as follows:

Table 8.1 Composition of the Sub-papers

Sub-paper	Number of Items (Score Points)			
	Number and Algebra Dimension	Measures, Shape and Space Dimension	Data Handling Dimension	Total
M1	20 (26)	23 (34)	6 (9)	49 (69)
M2	22 (30)	22 (32)	5 (7)	49 (69)
M3	24 (33)	20 (29)	5 (6)	49 (68)
M4	22 (32)	22 (31)	4 (5)	48 (68)
Total *	70 (93)	72 (103)	17 (23)	159 (221)

* Items that appeared in more than one sub-paper are counted only once.

The item types of the sub-papers were as follows:

Table 8.2 Item Types of the Sub-papers

Section	Percentage of Score Points	Item Types
A	~ 30%	<ul style="list-style-type: none"> Multiple-choice questions: choose the best answer from among four options
B	~ 30%	<ul style="list-style-type: none"> Calculate numerical values Give brief answers
C	~ 40%	<ul style="list-style-type: none"> Solve application problems showing working steps Draw diagrams or graphs Open-ended questions requiring reasons or explanations

Performance of S.3 Students with Minimally Acceptable Levels of Basic Competency in TSA 2009

S.3 Number and Algebra Dimension

The performances of S.3 students were satisfactory in this Dimension. In particular, they did better in items related to *Number and Number systems* and *Comparing Quantities*. Performance was only fair in items related to *Observing Patterns and Expressing Generality* and *Algebraic Relations and Functions*. Comments on students' performances are provided below with examples cited where appropriate (question number x / sub-paper y quoted as Q x /M y). More examples may also be found in the section ***General Comments***.

Number and Number Systems

- Directed Numbers and the Number Line: Students could handle the simple operation (involve one step only) of directed numbers generally. However, when the question contained the parentheses and more operations were required in solving the problem, some students could not perform the calculation correctly.
- Numerical Estimation: Students performed well in judging the reasonableness of answers from computations. Nevertheless, when they were required to estimate a value, many of them would use the calculator to find the answer directly and then transform the exact value into the estimated value. Besides, results were only fair when students had to justify their methods of estimation.

Q40/M3

Example of Student Work

(Estimation – (a) estimate the distance after the computation of the exact value;

(b) not able to explain the method of estimation.)

黃先生一週航行的總距離。
 $(4.9 + 12.85 + 9.3) \times 5$
 $= 27.05 \times 5$
 $\approx 135 \text{ km}$

∴ 黃先生一週航行 135 km。
我用四舍五入舍去兩款小數。

- Approximation and Errors: Some students were confused with the concepts of decimal places and significant figures. They didn't do well in representing a large or a small number in scientific notation. However, they were able to convert numbers in scientific notation to integers or decimals.

Q23/M4

Example of Student Work

(Using scientific notation to represent a number incorrectly)

起飛重量是 24×10^3 kg。

- Rational and Irrational Numbers: Students did quite well.

Comparing Quantities

- Using Percentages: (Percentage problem when expressed in $x \times y\% = z$ form) just about half of the students could solve x if only y and z were given. This does not indicate that they could not perform the division but, many students were confused with the meaning of each number in the question. However, students fared better on compound - interest problems than on simple - interest problems.

Q41/M2
Exemplar Item (Selling Problems in <i>Using Percentages</i>)
<i>The Student Council used 15% of school grant to buy a Ping Pong Table. The table cost \$1200. Find the amount of school grant.</i>
Example of Student Work (Taking school grant as 15% of the price of the table)
<p>學校交的撥款數是：</p> $1200 \times \frac{15}{100}$ $= 180 \text{ 元}$

- Rate and Ratio: Students did well. However, their performance was fair when they had to find the value of the quantity from a given ratio $a : b$.

Q24/M4
Exemplar Item (Finding other quantities from a given ratio $a:b:c$ and the value of a .)
<i>Given $a : b : c = 4 : 6 : 9$. If $a = 2$, find the values of b and c.</i>
Example of Student Work (Mistake: subtract 2 from all the original values)
$b = \underline{\quad 4 \quad} ; c = \underline{\quad 7 \quad} .$

Observing Patterns and Expressing Generality

- Formulating Problems with Algebraic Language: Students' performance was fair. When several conditions were given in the context, many students could not make use of them and formulated the equations incorrectly. Almost half of the students were not able to write the next few terms in a Fibonacci sequence. Generally, they were unfamiliar with the basic operation of number sequence.
- Manipulations of Simple Polynomials: Students' performance was satisfactory. They had knowledge of terminologies of polynomials and could do some basic manipulations with polynomials (including multiply a binomial by a binomial). However, they did not do well when asked to distinguish polynomials from algebraic expressions.

- Laws of Integral Indices: Students' performance was fair when they had to simplify algebraic expressions using laws of integral indices. There was room for improvement for some students on the simplification of expressions involving negative indices.

Q42/M4

Example of Student Work (Using the laws of integral indices wrongly)

$$\frac{x^5}{x^3 y^{-4}}$$

$$= \frac{x^8}{y^{-4}}$$

$$= x^8 y^4$$

Example of Student Work (Mistake occurred in dealing with the terms involving negative indices)

$$\frac{x^5}{x^3 y^{-4}}$$

$$= x^{5-3} y^4$$

$$= \frac{1}{x^2 y^4}$$

- Factorization of simple Polynomials: About half of the students could not demonstrate the recognition of factorization as a reverse process of expansion. Moreover, when students were asked to apply cross method to factorize expressions of the form $ax^2 + bx + c$, there was a great difference in their performances between the questions with $a=1$ and $a \neq 1$. Less than half of the students could factorize simple polynomials by taking out common factors correctly.

Q27/M4

Example of Student Work (Without factorizing polynomials by using the perfect square expressions)

$$4x^2 + 12x + 9 = \underline{4x(x+3) + 9}$$

Q28/M2

Example of Student Work (Cross method – careless computation)

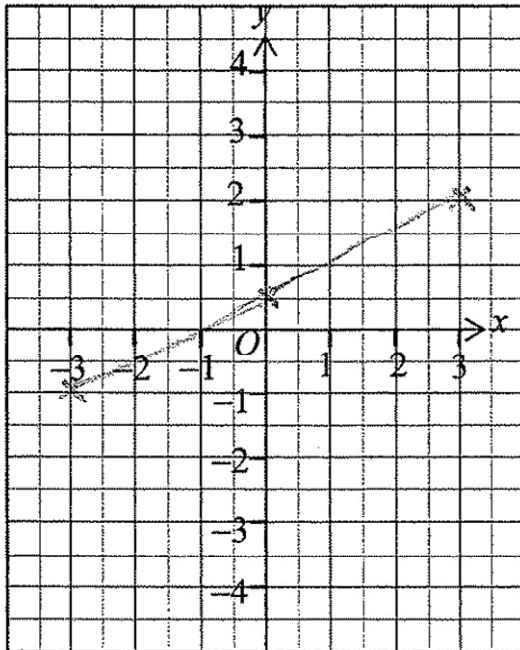
$$2x^2 - x - 6 = \underline{(x-1)(2x-6)}$$

Algebraic Relations and Functions

- Linear Equations in One Unknown: When the unknowns of the equation carried the negative signs, most of the students could not solve them correctly.
- Linear Equations in Two Unknowns: Students' performance was fair. They could use algebraic method and graphical method to solve linear simultaneous equations. Students in general could not plot graphs of linear equations without a hint. Moreover, when students tried to use algebraic methods to solve simultaneous equations, careless mistakes often appeared in the computation.

Q43/M3

Example of Student Work (Although the student was able to plot the graph, a ruler was not used)



Q43/M4

Example of Student Work

(Solving simultaneous equations – although students knew how to use the method of elimination, mistakes occurred in the computation)

$$\begin{cases} 2x - y = 78 & \text{--- ①} \\ 4x + y = 114 & \text{--- ②} \end{cases}$$

① $\times 2$

$$4x - 2y = 156 \quad \text{--- ③}$$

③ - ②

$$4x - 2y = 156 \quad \text{--- ③}$$
$$4x + y = 114 \quad \text{--- ②}$$
$$y = -6$$

Put $y = -6$ into ①

$$2x - y = 78$$
$$2x - (-6) = 78$$
$$x = 37 \quad \therefore x = 37, y = -6$$

Example of Student Work

(Solving simultaneous equations – although students knew how to use the method of substitution, mistakes occurred in dealing with the coefficients and constants)

$$x = 122 - x$$

a) ~~$\begin{cases} x + y = 122 & \text{--- ①} \\ 40x + 70y = 10200 & \text{--- ②} \end{cases}$~~

~~$10200 - 70y = 90x$~~

b) ~~② =~~

$$\frac{10200 - 70y}{40} = x \quad \text{--- ③}$$

③ into ①

$$y = 122 - \frac{10200 - 70y}{40}$$
$$40y = 122 - 10200 - 70y$$
$$160y = 122 - 10200$$
$$160y = -10078$$
$$y = -63$$

- Identities: Students were able to distinguish equations from identities. They fared better on using the difference of two squares than on the perfect square expressions to expand simple algebraic expressions, but performance was only fair in general.

Q29/M4
<p>Example of Student Work</p> <p>(Using difference of two squares in expansion – could not distinguish the difference between x^2 and $2x$)</p> $(x+3)(x-3) = \underline{2x-9.}$
<p>Example of Student Work (Using incorrect formula to expand the algebraic expressions)</p> $(x+3)(x-3) = \underline{x^2+6x-9}$

- Formulas: Students' performance was satisfactory. They could manipulate the algebraic fractions. However, when they were asked to change the variable in the denominator to be the subject of the formulas, their performance was unsatisfactory.

Q30/M2
<p>Exemplar Item (Change of subject)</p> <p>Make x the subject of the formula $w = \frac{10}{x} + 2$.</p>
<p>Example of Student Work (Change of subject - careless computation)</p> $\underline{\frac{10}{w+2}}$

- Linear Inequalities in One Unknown: Students' performance was fair. When the variable of the inequality carried the negative sign, most of the students could not solve it correctly.

S.3 Measures, Shape and Space Dimension

S.3 students performed quite well in this Dimension. They could solve basic geometric problems (such as finding measures in 2-D and 3-D figures, solving problems related to angles associated with lines and rectilinear figures, Pythagoras' Theorem, and simple application of trigonometry). However, more improvement could be shown in items related to reflectional symmetries and rotational symmetries of 3-D figures and geometric proofs. Comments on students' performances are provided below with examples cited where appropriate (question number x /sub-paper y quoted as Q x /M y). More items may also be found in the section *General Comments*.

Measures in 2-D and 3-D Figures

- Estimation in Measurement: Students did quite well, although some of them found difficulty in choosing an appropriate measuring tool and technique for real-life measurements.
- Simple Idea of Areas and Volumes: Students' performance was fair, only that they were negligent of the unit of the answer. In particular, performance was weak in application of the formulas for surface areas of cylinders.

Q45/M2
Example of Student Work (The radius of the circle was r cm, so r should bear no unit)
$\pi r^2 = 256\pi$
$r^2 = 256$
$r = 16 \text{ cm}$
$\text{Area} = 2\pi r$
$= 2 \times 16 \times \pi$
$= 32\pi \text{ cm}$

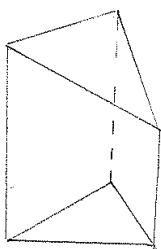
- More about Areas and Volumes: Students' performance was satisfactory. They could use formulas to compute measures generally, but had difficulties using relationships between sides and volumes of similar figures to solve problems.

Learning Geometry through an Intuitive Approach

- Introduction to Geometry: Students' performances were varied from person to person. They did well in problems relating to angles, but they didn't demonstrate recognition of regular polygon and Platonic Solids. Besides, more than half of the students were not able to sketch the triangular prism.

Q31/M4

Example of Student Work (Sketch of triangular prism – lateral plane was not the rectangle obviously)



Example of Student Work (Sketch of triangular prism – drawing lines were not clear)

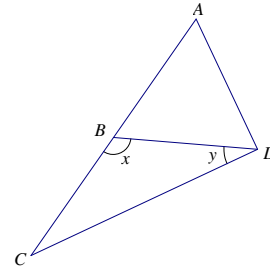


- Transformation and Symmetry: Students' performance was quite good. They demonstrated recognition of basic concepts. However, performance was weak in the questions about axes of symmetry and rotational symmetry.
- Congruence and Similarity: Students' performance was good.
- Angles related with Lines and Rectilinear Figures: Students did well. Most of them could solve simple geometric problems. However, some of the students just estimated sizes of angles and lengths of sides from the figure directly. They were not aware that in fact those data were not given in the question and the figures were not drawn in scale necessarily.

Q46/M1

Exemplar Item (Using the relations between sides and angles associated with isosceles/equilateral triangles)

In the figure, ABC is a straight line,
 $AB = BC = BD = AD$. Find x and y .



Example of Student Work (Treating $\angle ADC$ as 90°)

$$\triangle ABD = 180^\circ \text{ (全等, } AB = AD = BD)$$

$$\angle ABD = 180^\circ \div 3 \text{ (三角相等)}$$

$$= 60^\circ$$

$$x = 180^\circ - 60^\circ \text{ (直线上相邻角)}$$

$$= 120^\circ$$

$$y = 40^\circ - 60^\circ$$

$$= 30^\circ$$

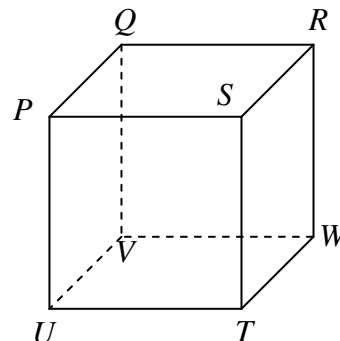
- More about 3-D figures: Same as last year, the majority of students could not name planes of reflectional symmetries or axes of rotational symmetries of cubes according to context of item. Although some of students could identify the plane, but they did not write down the name of plane in correct order (e.g. write $PWTQ$ instead of $PQWT$). They also did not do well when asked to name projection of edges on planes or angle between planes. However, they did well on items related to the nets of cubes and matching 3-D objects with various views.

Q35/M4

Exemplar Item (Name the appropriate plane of reflectional symmetry of a cube)

Only a few students answered correctly (answer: $PQWT$ or equivalent).

The figure shows a cube $PQRSTUW$. Using four of its vertices (i.e. P, Q, R, S, T, U, V or W), name **ONE** of the planes of reflectional symmetry of the cube.



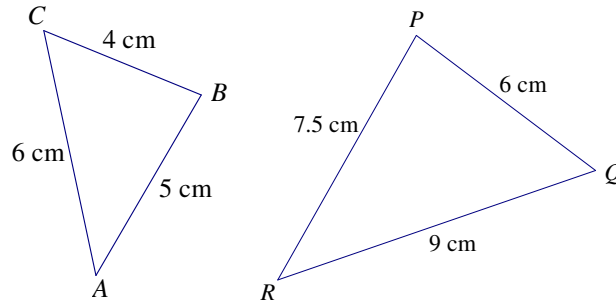
Learning Geometry through a Deductive Approach

- Simple Introduction to Deductive Geometry: More than half of the students could write some basic steps of a geometric proof, but most could not complete the proof correctly. Besides, many students were able to identify angle bisectors of a triangle.

Q48/M3

Exemplar Item (Geometric proof)

Refer to the figure. Prove that $\triangle ABC \sim \triangle RPQ$.



Example of Student Work (Using conditions for congruent triangles as conditions for similar triangles)

$$\frac{CB}{PQ} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{CA}{RQ} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{BA}{PR} = \frac{5}{7.5} = \frac{2}{3}$$

$$\frac{BA}{PR} = \frac{CA}{RQ} = \frac{CB}{PQ}$$

$\therefore \triangle ABC \sim \triangle RPQ$ (SSS)

Example of Student Work (Wrong logical proof – supporting argument and result were put in reverse order)

$$\frac{AP}{RP} = \frac{BC}{PQ} = \frac{AC}{RQ}$$

$$\frac{5}{7.5} = \frac{2}{3}$$

$$\frac{4}{6} = \frac{2}{3} \quad (\text{相似三角形的特性})$$

$$\frac{6}{9} = \frac{2}{3} \quad (\text{对应边成比例})$$

$\therefore \triangle ABC \sim \triangle RPQ$ (三边成比例)

Example of Student Work (Good performance)

$$\frac{QP}{CB} = \frac{6}{4} = 1.5$$

$$\frac{QR}{CA} = \frac{9}{6} = 1.5$$

$$\frac{PR}{BA} = \frac{7.5}{5} = 1.5$$

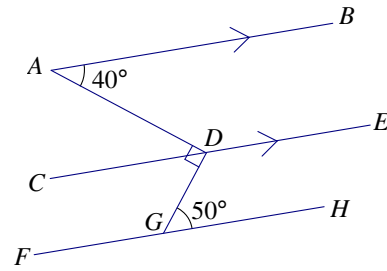
$\therefore \frac{QP}{CB} = \frac{QR}{CA} = \frac{PR}{BA} = 1.5$

$\therefore \triangle ABC \sim \triangle RPQ$ (三边成比例)

Q47/M2

Exemplar Item (Geometric proof)

In the figure, CDE and FGH are straight lines,
 $AB \parallel CE$, $\angle BAD = 40^\circ$, $\angle DGH = 50^\circ$,
 $\angle ADG = 90^\circ$. Prove that $CE \parallel FH$.



Example of Student Work (Without using 'alt. \angle s eq.' to prove $CE \parallel FH$)

$$\angle ADC = \angle BAD \text{ (錯角, } AB \parallel CE)$$
$$= 40^\circ$$

$$\text{又 } \angle ADG = 90^\circ \text{ (已知)}$$

$$\therefore \angle CDG = 90^\circ - 40^\circ$$
$$= 50^\circ$$

$$\therefore \angle CDG = \angle DGH = 50^\circ$$

$$\therefore \angle CDG \text{ 和 } \angle DGH \text{ 互為錯角}$$

$$CE \parallel FH.$$

- Pythagoras' Theorem: Most students could use Pythagoras' Theorem to solve simple problems. However, they were confused about the differences between Pythagoras' Theorem and its Converse.
- Quadrilaterals: Students performed well. They could use the properties of squares and rhombuses in numerical calculations.

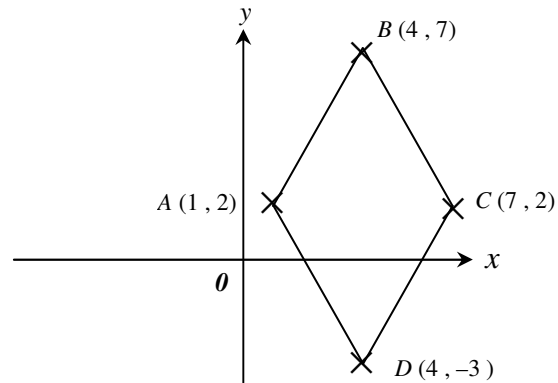
Learning Geometry through an Analytic Approach

- Introduction to Coordinates: Students' performance was fair. In particular, performance was weak when they were asked to calculate areas of simple figures that can be cut into or formed by common 2-D rectilinear figures.

Q36/M1

Exemplar Item (Calculating areas of simple figures)

Find the area of quadrilateral ABCD in the rectangular coordinate plane.



- Coordinate Geometry of Straight Lines: Students' performance was satisfactory. More than half of the students understood slopes of parallel lines were equal, but only some of them realized the relation of the slopes of perpendicular lines. Moreover, many students could not use the mid-point formula to find the coordinates of the mid-point of two given points.

Trigonometry

- Trigonometric Ratios and Using Trigonometry: Students showed certain degree of understanding in basic trigonometric ratios and applications. Most of them could find the sides when they were asked to solve right-angled triangles. However, only half of the students could find the angles in this type of problems.

Q47/M4

Example of Student Work (Solving the angles of a right-angled triangle – wrong presentation)

$$\begin{aligned} \text{該斜塔身與水平面間的角} &= \tan^{-1} \frac{53.86}{3.9} \\ &= 86^\circ \end{aligned}$$

Q47/M1

Example of Student Work (Solving the sides of a right-angled triangle – use trigonometric ratios and Pythagoras' Theorem to find the sides, but mistakes occurred in presentation)

$$X = \tan 47^\circ = \frac{X}{5}$$

$$X = \tan 47^\circ \times 5$$
$$= 5.36$$

$$Y^2 = 5.36^2 + 5^2$$

$$Y^2 = 53.73$$

$$Y = \sqrt{53.73}$$

$$Y = 7.33 \text{ cm}$$

S.3 Data Handling Dimension

S.3 students performed well in this Dimension. They showed recognition of various stages of Statistics. They also did well in items relating to interpreting simple statistical charts, comparing the presentations of the same set of data by using statistical charts and finding mean or median from a set of ungrouped data. There was great improvement in calculating the theoretical probability by listing. However, performance was weak when students were asked to distinguish discrete and continuous data and find mean from a set of grouped data. Comments on students' performance are provided below with examples cited where appropriate (question number x / sub-paper y quoted as Q x /M y). More examples may also be found in the section *General Comments*.

Organization and Representation of Data

- Introduction to Various Stages of Statistics: Students' performance was varied. Most of them could collect and organize data using simple methods. However, many students confused discrete data with continuous data.

Q19/M1

Exemplar Item (Distinguishing discrete and continuous data)

Which of the following is continuous data?

- A. The number of each kind of cookies in 10 boxes of cookies
- B. The lengths of 25 telephone ropes
- C. The sizes of 20 pairs of running shoes
- D. The marks of a Chinese quiz of 20 students

- Construction and Interpretation of Simple Diagrams and Graphs: Students did quite well. Most students could interpret simple statistical charts and identify sources of deception in misleading graphs. However, many of them could not construct stem-and-leaf diagrams correctly.

Q49/M2

Example of Student Work (Mistakes – add commas between the data)

15名學生擔任義工的時數	
幹(10小時)	葉(1小時)
1	3, 3, 5, 8
2	5, 5, 5, 5, 7
3	5, 6, 8
4	6
5	1, 1

Example of Student Work (Student didn't pay attention to the unit of stem)

15名學生擔任義工的時數	
幹(10小時)	葉(1小時)
10	3 3 5 8
20	5 5 5 5 7
30	5 6 8
40	6
50	1 1

Analysis and Interpretation of data

- Measures of Central Tendency: Most students could find the measures of central tendency from ungrouped data. However, most of them could not find such values when using grouped data.

Q39/M3

Exemplar Item (Find the arithmetic mean from a set of grouped data)

The Parent-teacher Association sold raffle tickets to parents attending Parents' Day. The results were as follows:

<i>Number of raffle tickets bought</i>	0 – 2	3 – 5	6 – 8
<i>Number of people</i>	31	67	2

What was the mean number of raffle tickets bought per person?

- Identify sources of deception in cases of misuse of averages: More than half of students could not explain the reasons clearly.

Q49/M3

Example of Student Work (Without explanation of the sources of deception)

有。因為他用了平均數來比較，應用中位數來比較。
因為中位數不會受數目大小而改變。

Q49/M3

Example of Student Work (Without explain that mean was easily affected by extreme values)

有。因為^他只把自己的分數相加，然後除以場數，
得出的結果是6.4，雖然高過5分，但他有些場
數只拿了3分和4分，因此這句說話有
誤導成分。

Q49/M3

Example of Student Work (Good performance)

有，
$$\frac{3+2+4+2+3+4+3+4+4+7+3}{10} = 6.7 \text{ 分}$$

他的平均數是6.7分，平均數會受偏大或偏小的數據
影響，他不能說經常在球賽中取得7分以上，他只有
2場以5分以上，另外8場是5分以下。
場

Probability

- Simple Idea of Probability: Most students could compute empirical probability. They showed a great improvement in computing a theoretical probability by listing.

Q39/M2

Exemplar Item (Calculating theoretical probability by listing)

Two fair coins are tossed once. Find the probability of getting one head and one tail.

General Comments on S.3 Student Performances

The overall performance of S.3 students was good. They did better in Data Handling Dimension. Performance was satisfactory in Number and Algebra Dimension and Measures, Shape and Space Dimension.

The areas in which students demonstrated adequate skills are listed below:

Directed Numbers and the Number Line:

- Use positive numbers, negative numbers and zero to describe situations in daily life (e.g. Q22/M2).
- Demonstrate recognition of the ordering of integers on the number line (e.g. Q21/M3).
- Add, subtract, multiply and divide directed numbers (e.g. Q21/M4).

Numerical Estimation:

- Determine whether to estimate or to compute the exact value in a simple context (e.g. Q22/M3).
- Judge, without actual calculations, the reasonableness of answers from computations (e.g. Q1/M1).

Rational and Irrational Numbers

- Demonstrate, without using calculators, recognition of the integral part of \sqrt{a} , where a is a positive integer not greater than 200 (e.g. Q1/M4).

Rate and Ratio

- Demonstrate recognition of the difference between rate and ratio (e.g. Q24/M1).

Formulating Problems with Algebraic Language

- Distinguish the difference between algebraic expressions (e.g. Q3/M4).
- Translate word phrases/contexts into algebraic languages (e.g. Q4/M1).
- Substitute values into some common and simple formulas and find the value of a specified variable (e.g. Q25/M3).

Linear Inequalities in One Unknown

- Formulate linear inequalities in one unknown from simple contexts (e.g. Q8/M2).

Estimation in Measurement

- Choose an appropriate unit and the degree of accuracy for real-life measurements (e.g. Q10/M2).

Introduction to Geometry

- Identify types of angles with respect to their sizes (e.g. Q13/M2).
- Make 3-D solids from given nets (e.g. Q13/M3).

Transformation and Symmetry

- Name the single transformation involved in comparing the object and its image (e.g. Q15/M4).

Congruence and Similarity

- Demonstrate recognition of the conditions for congruent and similar triangles (e.g. Q34/M1 and Q33/M4).

Angles related with Lines and Rectilinear Figures

- Demonstrate recognition of the angles associated with lines and their transversals (e.g. Q16/M2).
- Use the angle properties associated with intersecting lines/parallel lines to solve simple geometric problems (e.g. Q47/M3).

More about 3-D Figures

- Identify the nets of cubes (e.g. Q14/M4).
- Match 3-D objects built up of cubes from 2-D representations from various views (e.g. Q16/M1).

Pythagoras' Theorem

- Use Pythagoras' Theorem to solve simple problems (e.g. Q17/M3).

Quadrilaterals

- Use the properties of squares in numerical calculations (e.g. Q17/M4).

Introduction to Coordinates

- Use an ordered pair to describe the position of a point in the rectangular coordinate plane and locate a point of given rectangular coordinates (e.g. Q37/M3).

Introduction to Various Stages of Statistics

- Organize the same set of data by different grouping methods (e.g. Q49/M1).

Construction and Interpretation of Simple Diagrams and Graphs

- Choose appropriate diagrams/graphs to present a set of data (e.g. Q20/M3).
- Compare the presentations of the same set of data by using statistical charts (e.g. Q20/M4).
- Identify sources of deception in misleading graphs/accompanying statements (e.g. Q20/M1).

Other than items in which students performed well, the Assessment data also provided some entry points to strengthen teaching and learning. Items worthy of attention are discussed below:

Approximation and Errors

- Round off a number to a certain number of decimal places (e.g. Q2/M2): Some students were confused with the concepts of rounding off a number to a certain number of decimal places and significant figures, hence they chose D mistakenly.

Q2/M2

Round off 0.001 849 to 3 decimal places.

- A. 0.00
- B. 0.001
- C. 0.002
- D. 0.00 185

Formulating Problems with Algebraic Language

- Formulate simple equations/inequalities from simple contexts (e.g. Q5/M2): Less than half of the students chose the correct answer D. When a considerable number of conditions were given in the question, sometimes students did not realize the meaning of them in the problem. Therefore some students chose A mistakenly.

Q5/M2

The base fee of Helen's mobile phone plan is \$20. It includes 500 free minutes. The fee thereafter is \$0.1 per minute.

Helen used her mobile phone for more than 500 minutes this month. If she used the phone for x minutes, which of the following equations finds Helen's mobile phone fees \$ C this month?

- A. $C = 20 + (0.1)x$
B. $C = 20 + (0.1)x - 500$
C. $C = 20 + (500 - x)(0.1)$
D. $C = 20 + (x - 500)(0.1)$

Manipulations of Simple Polynomials

- Distinguish polynomials from algebraic expressions (e.g. Q4/M3): Another question related to the concept of polynomials as well, only about 40% of students chose the correct answer B. Some students thought that every term of a polynomial must contain the variable, hence they chose the wrong answer A.

Q4/M3

Which of the following is a polynomial in x ?

- A. $x^2 + 2x + \frac{1}{x}$
B. $x^2 + 2x + \frac{1}{2}$
C. $\frac{1}{x^2 + 2x + 1}$
D. $x^2 + 2\sqrt{x} + 1$

- Demonstrate recognition of terminologies of polynomials (e.g. Q5/M1): There was room for improvement for students on the concept about polynomials. Some students thought that the terms with same coefficients were the like terms and chose the wrong answer D.

Q5/M1

Which of the following polynomials has two like terms?

- A. $3x - 3y$
- B. $3x - 5x$
- C. $5x + 3x^2$
- D. $7x + 7xy$

- Add or subtract polynomials of at most 4 terms: If the indices of the terms in polynomials were not all equal (e.g. Q26/M3), the performance of students was weak and the majority of them could not find the correct answer.

Q26/M3

Simplify $(x^3 + 5x) - (x^2 - 3x)$.

- Even when the indices of all the terms were equal (e.g. Q6/M2), some students still added the indices of the terms together mistakenly and chose the wrong answer C.

Q6/M2

Simplify $3x^3 + 2x^3$.

- A. $5x^3$
- B. $6x^3$
- C. $5x^6$
- D. $6x^6$

Linear Equations in Two Unknowns

- Plot graphs of linear equations in 2 unknowns: Two different items were set in the Assessment in different sub-papers. One of the items provided a table preset with some coordinates to assist plotting. The other item asked students to plot directly.

Q43/M2

Complete the following table for the equation $2y = x + 1$ in the ANSWER BOOKLET:

x	-3	0	3
y			2

Draw the graph of this equation on the rectangular coordinate plane given in the ANSWER BOOKLET.

Q28/M1

Draw the graph of $2y = x + 1$ on the given rectangular coordinate plane in the ANSWER BOOKLET.

- Students did well in Q43/M2. However, the facility of Q28/M1 was only about half that of Q43/M2.
- Determine whether a point lies on a straight line with a given equation (e.g. Q7/M1): About half of the students chose the correct answer C. Many students did not match the values of x and y correctly and chose B consequently.

Q7/M1

Given that the equation of straight line L is $x - 2y + 2 = 0$. Which of the following points lies on L ?

- A. (2, 1)
- B. (-2, 1)
- C. (198, 100)
- D. (-198, 100)

Identities

- Tell whether an equality is an equation or an identity (e.g. Q9/M3): More than half of the students chose the correct answer D, but some students could not understand the difference between equation and identity.

Q9/M3

Which of the following is an identity?

- A. $x^2 = 0$
- B. $3x = 6$
- C. $3x - 2 = 2 - 3x$
- D. $2(x + 3) - 2 = 2(x + 2)$

Estimation in Measurement

- Estimate measures with justification (e.g. Q44/M1): Although students knew how to estimate the height of the building in general, many of them could not justify their methods.

Q44/M1

a. 估計建築物 16 m

b. 因為男孩的身高等於窗的一半高

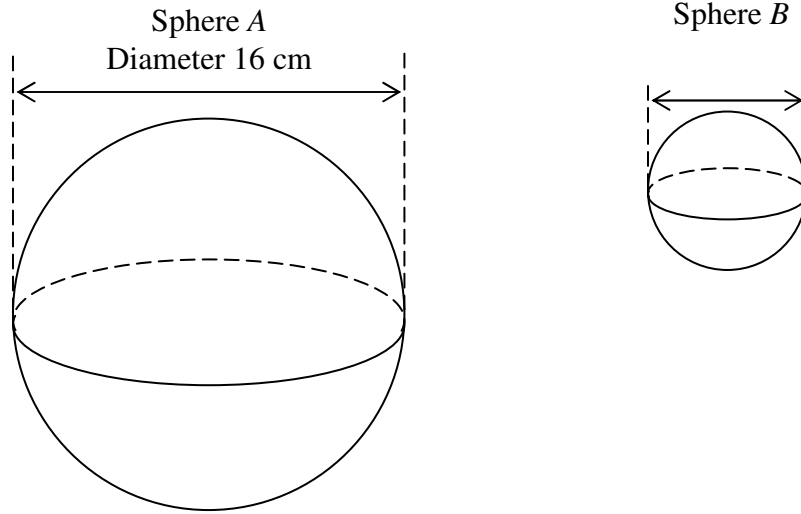
More about Areas and Volumes

- Use the relationships between sides and surface areas/volumes of similar figures to solve related problems (e.g. Q11/M3): Almost half of the students treated the relationship between sides the same as relationship between volumes (option B). Only a few students answered correctly (option D).

Q11/M3

In the figure, the volume of sphere A is 8 times that of sphere B. The diameter of sphere A is 16 cm. Find the diameter of sphere B.

- A. 1 cm
- B. 2 cm
- C. 4 cm
- D. 8 cm

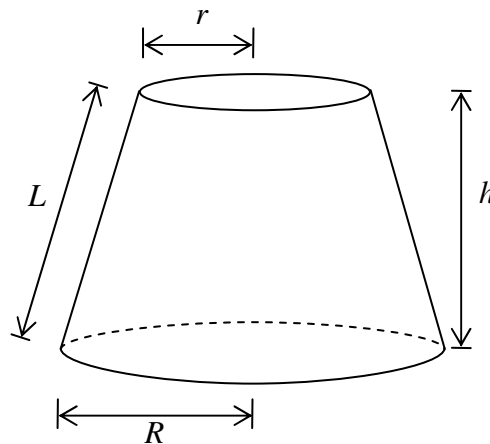


- Distinguish among formulas for lengths, areas and volumes by considering dimensions (e.g. Q11/M4): Only some students could chose the correct answer B. Almost half of the students took the formula in option C, which was similar to the formula for volume of circular cones to be the answer.

Q11/M4

The frustum in the figure is formed by a right circular cone cutting off its top. Its top radius and base radius are r and R respectively. Its height is h , and lateral height is L . Considering dimensions, determine which of the following could be the formula of lateral surface area of the frustum.

- A. $\pi(r + R)$
- B. $\pi(r + R)L$
- C. $\frac{1}{3}\pi h(R^2 + r^2 + Rr)$
- D. $\pi(R + r + L + h)$

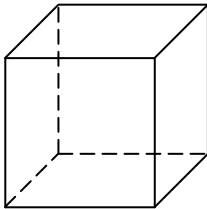
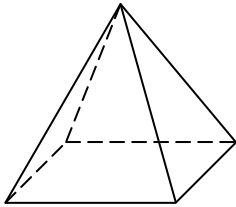
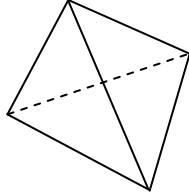
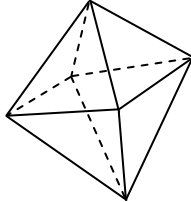


Introduction to Geometry

- Demonstrate recognition of common terms in geometry (e.g. Q13/M1): Only a small number of students could identify the 3-D figure in option B was not a regular polyhedron. About half of the students chose C mistakenly.

Q13/M1

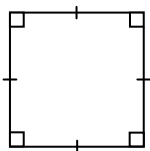
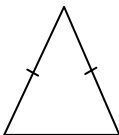
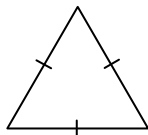
The followings are diagrams of four 3-D figures. Which of these CANNOT be a regular polyhedron?


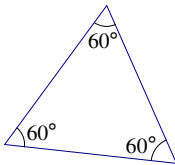
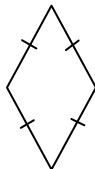
A.  B.  C.  D. 

- Determine whether a polygon is regular (e.g. Q31/M3): Most students thought that the polygon in F was regular. Only a few students chose A, C and E as the correct answers.

Q31/M3

Which of the following figures must be regular polygon(s)?

A.  B.  C. 

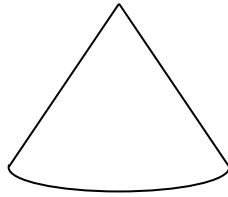
D.  E.  F. 

- Sketch cross-sections of simple solids (e.g. Q13/M4): Some students didn't see the correct shape of cross-section of the cone and chose A as the answer. Only half of the students chose B correctly.

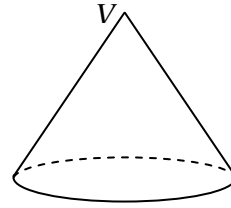
Q13/M4

The figure shows a right circular cone. Ken needs to draw a cross-section which is perpendicular to the base and passing through vertex V. The plane diagram of the cross-section is

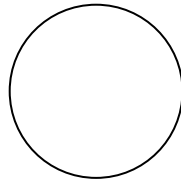
A.



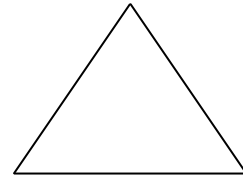
B.



C.



D.

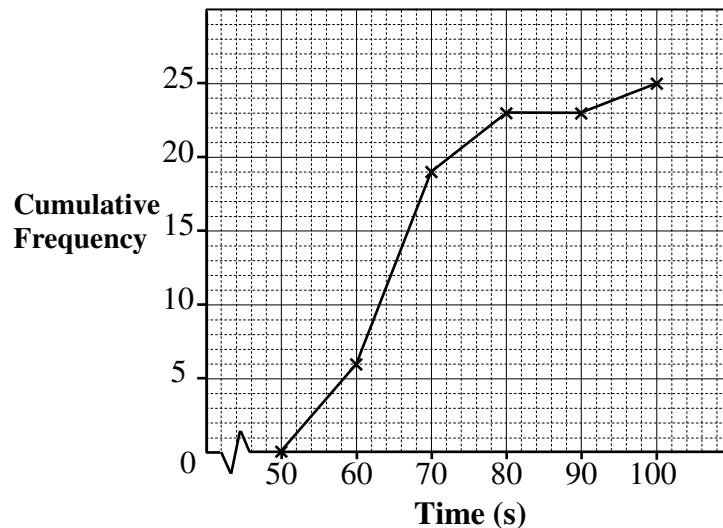


Construction and Interpretation of Simple Diagrams and Graphs

- Read information (including percentiles, quartiles, median) and frequencies from diagrams/graphs (e.g. Q40/M2): Less than half of the students could provide the correct answer (6 students). Some of them took the number of students who finished the race at 70 s (19 students) to be the answer.

Q40/M2

The following cumulative frequency polygon shows the 400 m race running times of 25 students:



How many students used more than 70 s to finish the race?

Best performance of S.3 Students in TSA 2009

Students were ranked according to their scores and the performances of the top 10% were analyzed further.

Most of these students either achieved the full maximum score or lost one or two score points in the Assessment. They demonstrated almost complete mastery of the concepts and skills assessed by the sub-papers attempted.

Most of these students were able to add, subtract, multiply and divide directed numbers (e.g. Q21/M4), translate word phrases/contexts into algebraic languages (e.g. Q4/M1), solve a system of simple linear simultaneous equations (e.g. Q44/M2), demonstrate recognition of the ideas of bearing, gradient, the angle of elevation and the angle of depression (e.g. Q48/M1), use the angle properties associated with intersecting lines/parallel lines to solve simple geometric problems (e.g. Q47/M3) and use the properties of parallelograms, squares, rectangles, rhombuses, kites and trapeziums in numerical calculations (e.g. Q17/M4).

Students with the best performance could solve simple selling problems (e.g. Q40/M4) and use the formulas for circumferences and areas of circles (e.g. Q45/M2).

Q44/M1

Example of Student Work (Estimation): According to the context given by the question, justified the method of estimation clearly.

(a) 估計建築物有 12.8 m.

(b) 可估計到小男孩大約有 4 個才可以是大樹的高度，
而建築物則大約有 2 顆大樹的高度，所以估計有
8 個小孩的高度便等於建築物的高度。

Q43/M1

Example of Student Work (Find the area of sector): Solved the problem correctly and rounded off the answer to a certain number of decimal places as required by the question.

該扇形的面積：
 $\pi (8)^2 \times \frac{60}{360}$
 $= 64\pi \times \frac{60}{360}$
 $= 33.5 \text{ cm}^2$ (取至最接近的 0.1 cm^2)

Q44/M2

Example of Student Work (Formulated and solved simultaneous equations):

Formulated simultaneous equations from the given context and solved them by the method of substitution.

$$\begin{aligned} & \begin{cases} 90x + 70y = 10200 \\ x + y = 122 \end{cases} \\ & \text{b. 2. } \begin{cases} 90x + 70y = 10200 \quad \text{--- (1)} \\ x + y = 122 \quad \text{--- (2)} \end{cases} \\ & \quad x = 122 - y \quad \text{--- (3)} \\ & \text{1. (1) } \times \text{ (3)} \\ & \quad 90(122 - y) + 70y = 10200 \\ & \quad 10980 - 90y + 70y = 10200 \\ & \quad -90y + 70y = 10200 - 10980 \\ & \quad -20y = -780 \\ & \quad 20y = 780 \\ & \quad y = 39 \\ & \quad \therefore \text{ 每天快车的载客量是 } 39 \text{ 位乘客。} \end{aligned}$$

Q47/M2

Example of Student Work (Geometric proof): Showed steps clearly and used correct reasoning to set up the conclusion.

$$\begin{aligned} & \because AB \parallel CE \\ & \therefore \angle BAD + \angle ADE = 180^\circ \text{ (Int. } \angle\text{s, } AB \parallel CE) \\ & \quad \angle ADE = 140^\circ \\ & \quad \angle ADC + \angle ADE = 180^\circ \text{ (adj. } \angle\text{s on st. line)} \\ & \quad \angle ADC = 40^\circ \\ & \because \angle ADG = 90^\circ \text{ (given)} \\ & \therefore \angle CDG = 90^\circ - 40^\circ = 50^\circ \\ & \quad = \angle DGH \\ & \therefore CE \parallel FH \text{ (alt. } \angle\text{s, equal)} // \end{aligned}$$

Some common weak areas of high-achieving students are listed as follows:

- Some students could not distinguish polynomials from algebraic expressions.
- Some students could not fully understand the definition of regular polyhedron.
- Some students could not construct the stem-and-leaf diagrams correctly.
- Some students could not find mean from a set of grouped data.
- Many students could not determine whether a polygon is regular.
- Many students could not distinguish discrete and continuous data.

Comparison of Student Performances in Mathematics at Secondary 3 TSA 2007, 2008 and 2009

This was the fourth year that Secondary 3 students participated in the Territory-wide System Assessment. The percentage of students achieving Basic Competency in this year was 80.0% which was slightly higher than the percentage for the last year by 0.2%. It was also the highest percentage across the years.

The percentages of students achieving Basic Competency from 2007 to 2009 are listed below:

Table 8.3 Percentages of S.3 Students Achieving Mathematics Basic Competency from 2007 to 2009

Year	% of Students Achieving Mathematics Basic Competency
2007	79.9
2008	79.8
2009	80.0

The performances of S.3 students over the past three years in each Dimension of Mathematics are summarized below:

Number and Algebra Dimension

- Directed Numbers and the Number Line: The performance of students remained steady over the past three years. Students performed slightly better in four basic operations of directed numbers.
- Numerical Estimation: Students showed significant improvement in judging the reasonableness of answers from computations. However, students found difficulty with items requiring estimation and explanation.
- Approximation and Errors: Performance of students improved on problems requiring conversion of significant figures.
- Rational and Irrational Numbers: Performance remained steady. The performance of students was better in the usage of number line.
- Using Percentages: There was room for improvement particularly with problems related to simple-interest where their performance declined.

- Rate and Ratio: Performance remained steady.
- Formulating Problems with Algebraic Language: The performance of students declined in formulating equations from simple contexts. Finding intuitively the n^{th} term of a simple number sequence and finding the terms of the sequence from a given n^{th} term were still the weak spots.
- Manipulations of Simple Polynomials: Many students failed to distinguish polynomials from algebraic expressions and add or subtract polynomials. However, they did better in multiplying a binomial by a binomial.
- Laws of Integral Indices: Performance remained steady. Nevertheless, many students were not able to solve the problems involving negative indices.
- Factorization of Simple Polynomials: Performance was fair. Students had difficulty in factorizing simple polynomials by taking out common factors or grouping terms. They were not capable of using the cross method to factorize expressions. However, they made a slight improvement in items using perfect square expressions to factorize simple polynomials.
- Linear Equations in One Unknown: Performance remained steady. They regressed on items relating to solve simple equations.
- Linear Equations in Two Unknowns: Performance remained steady. Same as last year, most students still needed help in order to plot graphs of linear equations.
- Identities: Performance remained fair. Students made an improvement in items requiring the difference of two squares and perfect square expressions.
- Formulas: Performance declined when students were asked to perform a change of subject in simple formulas but not including radical sign.
- Linear Inequalities in One Unknown: There was a trend of decline in students' performance when using inequality signs \geq , $>$, \leq and $<$ to compare numbers. The weakness of students was still in solving inequalities.

Measures, Shape and Space Dimension

- Estimation in Measurement: Performance of students clearly declined in items involving more judgments (such as choosing an appropriate measuring tool and technique, reducing errors in measurements). Their performance was steady in

items relating to estimate measures with justification.

- Simple Idea of Areas and Volumes: Performances of students remained steady. They were able to apply formulas for volumes of prisms and cylinders. Moreover, they performed better in problems involving area of circle.
- More about Areas and Volumes: Students made continuous improvement. They were able to calculate the areas and volumes of various figures (e.g. volumes of pyramids) in general. However, there was room for improvement in items dealing with relationships of sides and volumes in similar figures.
- Introduction to Geometry: Students' performance was fair. They were weak in recognition of common terms in geometry (e.g. regular polygons and regular polyhedron). Most of them failed to sketch simple solids. But they performed well when they were required to make 3-D solids from given nets.
- Transformation and Symmetry: Performances of students declined slightly. They showed an inadequate grasp of some mathematical concepts (such as axes of symmetry and order of rotational symmetry).
- Congruence and Similarity: Students performed better this year. They improved in recognition of the conditions of congruence and of similarity.
- Angles related with Lines and Rectilinear Figures: Performances remained steady. They showed improvement in recognition of the terminologies on angles.
- More about 3-D Figures: Students' performance varied with respect to different BC descriptors within this unit. They could identify the nets of solids and match 3-D objects from 2-D representations. However, performance was still weak when dealing with the angles, lines, and planes associated with 3-D figures.
- Simple Introduction to Deductive Geometry: Performance of students was fair. As in past three years, students were willing to attempt the geometric proofs. Nevertheless, they usually could not give logical steps and correct reasons to complete the proofs.
- Pythagoras' Theorem: Performances remained steady. Many students were not capable of using the converse of Pythagoras' Theorem to solve simple problems.

- **Quadrilaterals:** Students' performance improved. Students could use the properties of the figures in numerical calculations in general.
- **Introduction to Coordinates:** Students' performance remained steady. However, they regressed significantly on items relating to calculate areas of simple figures that can be cut into or formed by common 2-D rectilinear figures.
- **Coordinate Geometry of Straight Lines:** Students' performance remained steady. They regressed slightly on the application of mid-point formula.
- **Trigonometric Ratios and Using Trigonometry:** Performances remained steady in general. Students showed improvement in items involving angle of depression and bearing.

Data Handling Dimension

- **Introduction to Various Stages of Statistics:** Students showed improvement in this unit, especially in problems related to organize the same set of data by different grouping methods. However, their performance was still unsatisfactory when distinguishing between discrete and continuous data.
- **Construction and Interpretation of Simple Diagrams and Graphs:** Students showed improvement in interpreting simple statistical charts, choosing appropriate diagrams, comparing the presentations of the same set of data by using statistical charts and identifying sources of deception. Same as in past years, they were weak in constructing statistical charts.
- **Measures of Central Tendency:** Performances remained steady in general. However, most students were not able to find median and mean from a set of grouped data.
- **Simple Idea of Probability:** Performance of students was satisfactory or above average. There was an improvement in items requiring calculation of theoretical probability by listing.