Results of Secondary 3 Mathematics in TSA 2008

The territory-wide percentage of S.3 students achieving Mathematics Basic Competency in TSA 2008 was 79.8%. In 2007 the percentage was 79.9%.

Secondary 3 Assessment Design

The design of assessment tasks for S.3 was based on the documents *Mathematics Curriculum: Basic Competency for Key Stage 3 (Tryout Version)* and *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5), 1999*. The tasks covered the three dimensions of the mathematics curriculum, namely **Number and Algebra**, **Measures**, **Shape and Space**, and **Data Handling**. They focused on the Foundation Part of the S1 – 3 Whole Syllabus in testing of the relevant concepts, knowledge, skills and applications.

The Assessment consisted of various item types including multiple-choice questions, fill in the blanks, answers-only questions and questions involving working steps. The item types varied according to the contexts of the questions. Some test items consisted of sub-items. Besides finding the correct answers, students were also tested in their ability to present solutions to problems. This included writing out the necessary statements, mathematical expressions and explanations.

The Assessment consisted of 170 test items (211 score points), covering all of the 129 Basic Competency Descriptors. These items were organized into four sub-papers, each 65 minutes in duration and covering all three Dimensions. Some items appeared in more than one sub-paper to act as inter-paper links. Each student was required to attempt one sub-paper only.

The composition of the sub-papers was as follows:

Table 8.5 Composition of the Sub-papers

Sub-paper	Number of Items (Score Points)			
	Number and Algebra Dimension	Measures, Shape and Space Dimension	Data Handling Dimension	Total
M1	26 (33)	23 (29)	7 (8)	56 (70)
M2	26 (33)	23 (28)	5 (9)	54 (70)
M3	26 (31)	24 (31)	6 (8)	56 (70)
M4	24 (30)	23 (31)	6 (10)	53 (71)
Total *	75 (90)	76 (96)	19 (25)	170 (211)

* Items that appeared in more than one sub-paper are counted only once.

The item types of the sub-papers were as follows:

Table 8.6 Item Types of the Sub-papers

Section	Percentage of Score Points	Item Types
A	~ 30%	• Multiple-choice questions: choose the best answer from among four options
B	~ 40%	• Calculate numerical values • Give brief answers
C	~ 30%	• Solve application problems showing working steps • Draw diagrams or graphs • Open-ended questions requiring reasons or explanations

Performance of S.3 Students with Minimally Acceptable Levels of Basic Competency in TSA 2008

S.3 Number and Algebra Dimension

Students did well in this Dimension. In particular, they did better in items related to *Number and Number systems* and *Comparing Quantities*. Performance was only satisfactory in items related to *Observing Patterns and Expressing Generality* and *Algebraic Relations and Functions*. Comments on students' performances are provided below with examples cited where appropriate (question number *x* / sub-paper *y* quoted as Q*x*/M*y*). Other items worthy of attention may also be found in the section ***General Comments*.**

Number and Number Systems

- Directed Numbers and the Number Line: Students did well. There was room for improvement on operations of directed numbers.
- Numerical Estimation: Students could do simple estimation in general. However, results were only fair when students had to justify their methods of estimation or judge reasonableness of estimated answers.

Q47/M3
Example of Student Work (Estimation – not able to explain unreasonableness of estimated value) 我認為不合理，因為60cm和5.4cm是實質數目，不能用四捨五入計算

- Approximation and Errors: Only about half of the students could convert significant figures correctly. There was also room for improvement in using scientific notation.

Q23/M1
Example of Student Work (Using scientific notation to represent a very small number - confuses use of symbols) 0.000 000 023 5 = $2.35 \cdot 10^{-8}$

- Rational and Irrational Numbers: Students did quite well. They had good knowledge of the integral part of $\sqrt{a}$.

Comparing Quantities

- Using Percentages: Only a small number of students could solve simple selling problems correctly. Moreover, students fared much better on simple-interest problems than on compound-interest problems.

Q24/M2
Exemplar Item (Selling Problems in *Using Percentages*) *An old model camera was sold for $1800. The percentage loss was 10%. Find the cost of the camera.*
Example of Student Work (Finding the cost by adding 10% to the selling price) 這部相機的成本是 $ 1980 。

Example of Student Work (Finding the cost by reducing selling price by 10%)

The cost of the camera was $ 1620.

Q46/M4

Exemplar Item (Problems on Growths in *Using Percentages*)

Henry bought a gold watch for $50 000 three years ago. Its value increased by 10% each year. Find the present value of the gold watch.

Example of Student Work (good performance)

金錶現時的價值：

$50\,000 \times (1+10\%)^3$

$= 66550$ 元

Q50/M1

Exemplar Item (Compound-interest Problem in *Using Percentages*)

Donald deposits $ 30 000 in a bank for 2 years. The interest rate is 4% p.a. compounded yearly. Find the total interest that Donald will receive.

Example of Student Work (correct solution)

$\$30\,000 \times (1+4\%)^2$

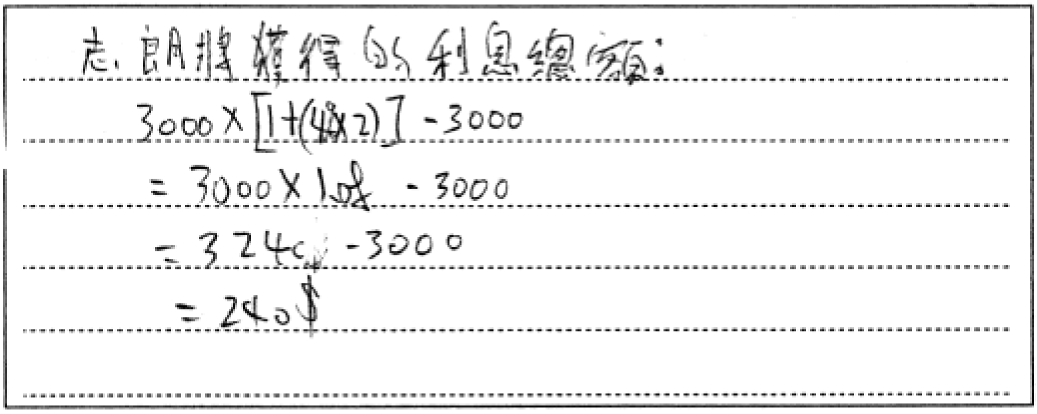
$= \$30000 \times 1.04^2$

$= \$\ 32440$

∴ 利息總額 $= 32440 - 30000 = \$2448$

Example of Student Work

(incorrectly using the method of simple interest instead of compound interest)


志朗將獲得的利息總額：

3000 × [1+(4%×2)] - 3000

= 3000 × 1.08 - 3000

= 3240 - 3000

= 240$

- Rate and Ratio: Students did well. However, there was room for improvement when students had to represent a ratio in the form $a : b$.

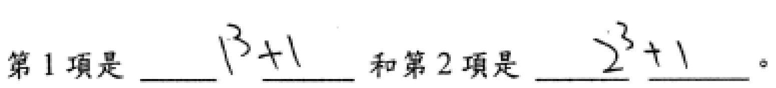
Observing Patterns and Expressing Generality

- Formulating Problems with Algebraic Language: Students did well. Most were able to do basic manipulations with formulas and sequences.

Q27/M3

Example of Student Work

(Sequences – substituting values without calculation when finding values of terms of sequence)


第 1 項是 1^3+1 和第 2 項是 2^3+1 。

- Manipulations of Simple Polynomials: Students' performances were fair. They had knowledge of terminologies of polynomials and could do some basic manipulations with polynomials. However, they did not do well when asked to multiply a binomial by a binomial.

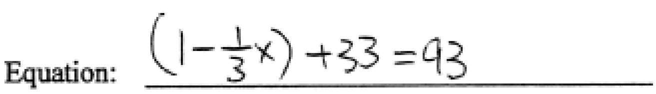
Q29/M3
Example of Student Work (multiply a binomial by a binomial – careless computation) 

- Laws of Integral Indices: Students' performances were only fair when they had to simplify algebraic expressions using laws of integral indices.
- Factorization of simple Polynomials: Students did not do well using cross method and using difference of two squares or the perfect square to factorize polynomials.

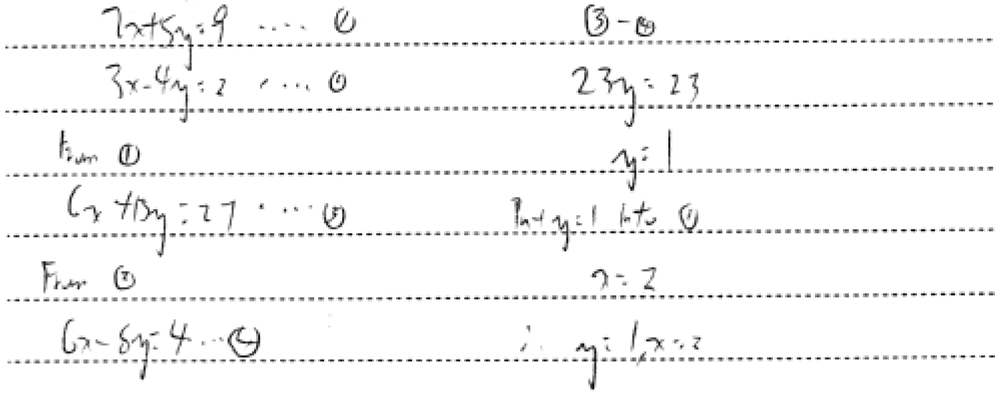
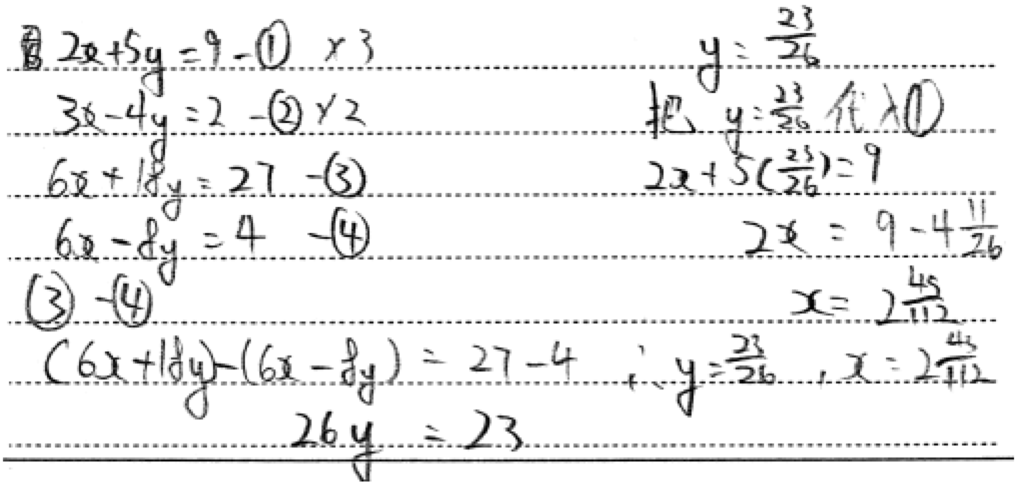
Q32/M1
Example of Student Work (cross method – careless computation) 

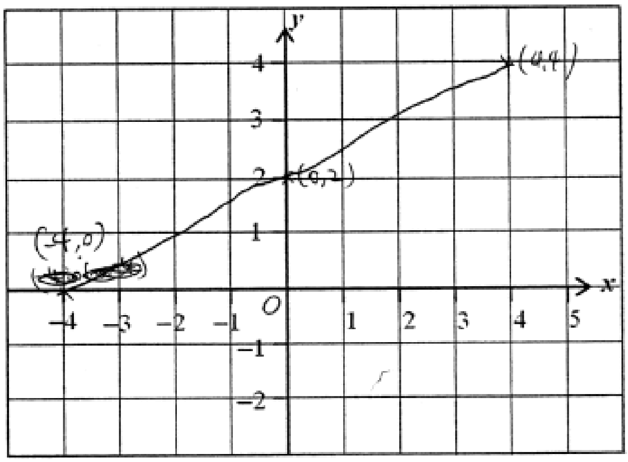
Algebraic Relations and Functions

- Linear Equations in One Unknown: Students could solve simple equations. Performances were weaker in formulating equations from given contexts.

Q33/M1
Example of Student Work (Formulating equations – careless use of parentheses: correct answer is $(1-\frac{1}{3})x+33=93$ or equivalent) 

- Linear Equations in Two Unknowns: Students' performances were fair. They could use algebraic methods to solve linear simultaneous equations, but did not do well when using graphical method. Students in general could not plot graphs of linear equations without a hint. Moreover, when students tried to use algebraic methods to solve simultaneous equations, careless mistakes often appeared in the computation.

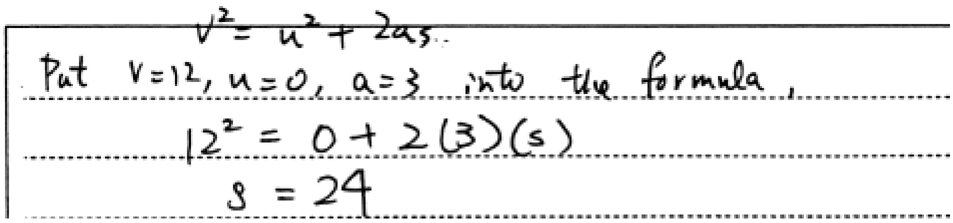
Q50/M2
Example of Student Work (Solving simultaneous equations – correct solution) 
Example of Student Work (Solving simultaneous equations – although knew how to solve equations, a mistake occurred in the computation) 

Q49/M2
Example of Student Work (Graphing linear equation – although the student was able to plot the graph, a ruler was not used) 

- Identities: Students were able to distinguish equations from identities, but performances were only fair in expansion of algebraic expressions using difference of two squares expressions.

Q34/M2
Example of Student Work (Using difference of two squares in expansion – not completing calculation) $(2x-5y)(2x+5y) = (2x)^2-(5y)^2$
Example of Student Work (Using difference of two squares in expansion – using incorrect formula) $(2x-5y)(2x+5y) = 4x^2+25y^2$
Example of Student Work (Using difference of two squares in expansion – not using parentheses correctly) $(2x-5y)(2x+5y) = 2x^2-5y^2$

- Formulas: Students did well. However, there was room for improvement in manipulation of algebraic fractions.

Q52/M1
Example of Student Work (Substituting values of formulas and find the unknown – good performance) 

- Linear Inequalities in One Unknown: Students' performances were fair though they could improve on solving linear inequalities.

S.3 Measures, Shape and Space Dimension

S.3 students performed satisfactorily in this Dimension. They could solve basic geometric problems (such as using formulas to find measures, solving problems related to angles associated with lines and rectilinear figures, Pythagoras' Theorem, and simple application of trigonometry). However, more improvement could be shown in items related to 3-D figures and geometric proofs. Comments on students' performances are provided below with examples cited where appropriate (question number *x* /sub-paper *y* quoted as Q*x*/M*y*). Other items worthy of attention may also be found in the section ***General Comments***.

Measures in 2-D and 3-D Figures

- Estimation in Measurement: Students did quite well. They showed improvement in items related to estimating measures and explanations.
- Simple Idea of Areas and Volumes: Students' performances were fair. In particular, performance was weak in application of circumferences of circles.
- More about Areas and Volumes: Students' performances were fair. Most students could use formulas to compute measures, but had difficulties using relationships between sides and volumes of similar figures to solve problems.

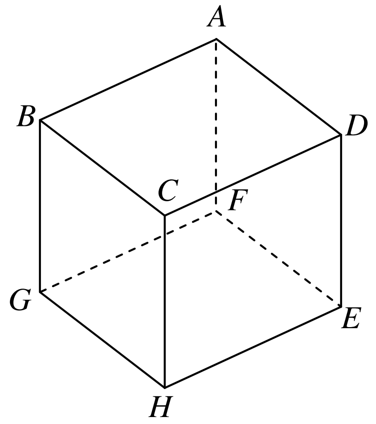
Q53/M1
Example of Student Work (Using formula to find surface area of cone – good performance) The total surface area of the cone $= 2(\pi)(6) + \pi(6)(10)$ $= 72\pi \text{ cm}^2$

Q53/M3
Example of Student Work (Using formula to find area of sector – good performance) $\left(\pi(1.4)^2 \times \frac{300^\circ}{360^\circ}\right)$ m $= \left[\pi 1.96 \times \frac{5}{6}\right]$ m $= 5.1$ m

Learning Geometry through an Intuitive Approach

- Introduction to Geometry: Students did well. Students improved in recognizing the cross-section of simple solids, but did not do well when asked to identify convex polygons.
- Transformation and Symmetry: Students' performances were satisfactory. They showed a grasp of basic concepts. However, performances were weak in distinguishing reflective transformation and rotational transformation.
- Congruence and Similarity: Students' performances were fair. Sometimes they were confused about conditions of congruent and similar triangles.
- Angles related with Lines and Rectilinear Figures: Students did well. The majority could solve simple geometric problems.
- More about 3-D figures: Most students could not name planes of reflectional symmetries of cubes according to context of item. They also did not do well when asked to name projection of edges on planes or angle between planes. However, they did well on items related to axes of rotational symmetries of cubes, nets, and matching 3-D objects with various views.

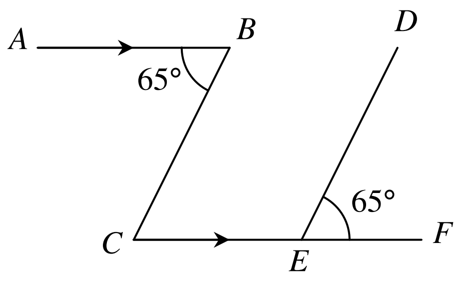
Q40/M3
Exemplar Item (Name the appropriate plane of reflectional symmetry of a cube) Only a few students answered correctly (answer: *BGED* or equivalent). *The figure shows a cube ABCDEFGH. Name the plane of reflectional symmetry containing vertices B and G.*



Learning Geometry through a Deductive Approach

- Simple Introduction to Deductive Geometry: About half of the students could write some basic steps of a geometric proof, but most could not complete the proof correctly. Moreover, some students were confused with the altitude and perpendicular bisector of triangles.

Q54/M3
Exemplar Item (Geometric proof) *In the figure, ∠ABC = ∠DEF = 65°, CEF is a straight line and AB // CF. Prove that BC // DE.*



Example of Student Work

(Did not use "Corresponding angles equal" to show two lines parallel)

∠ABC = ∠DEF = 65° (已知)

∵ ∠ABC = ∠BCE = 65° (內錯角, AB // CE)

∠BCE = ∠DEF = 65° (同位角, BC // DE)

∴ BC // DE.

Example of Student Work

(The notation ∠E could cause confusion, and correct reasons were not stated)

證明 BC // DE

∠B = ∠C 內錯角

∠C = ∠E 同位角

∴ BC // DE

Example of Student Work (good performance)

AB // CF 已知。

∵ AB // CF

∴ ∠BCE = 65° (內錯角 AB // CF)

∵ ∠DEF = ∠BCE = 65°

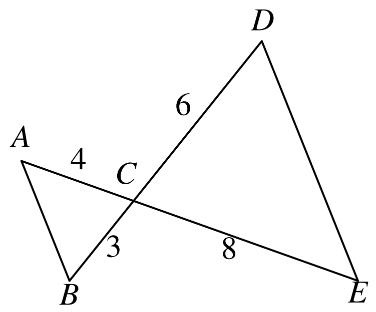
∴ BC // DE (同位角相等)。

Q52/M2

Exemplar Item (Geometric proof)

In the figure, ACE and BCD are straight lines.
AC = 4, BC = 3, CD = 6 and CE = 8 .

Prove that ΔABC ~ ΔEDC.



Example of Student Work

(Using conditions for congruent triangles as conditions for similar triangles)

$\because AC : EC = 1:2$

$BC : DC = 1:2$

$\angle ACB = \angle ECD$ (對頂角)

$\therefore \triangle ABC \sim \triangle EDC$ (SAS)

Example of Student Work

(Concept of proof not clear and confusing "3 sides proportional" and "two sides proportional with included angle")

In $\triangle ABC$ & $\triangle EDC$,

$\frac{AC}{CE} = \frac{4}{8} = \frac{1}{2}$

$\frac{BC}{CD} = \frac{3}{6} = \frac{1}{2}$

$\because AC = CE$ & $BC : CD = \frac{1}{2}$

$\therefore \frac{AB}{DE} = \frac{1}{2}$

$\therefore \triangle ABC \sim \triangle EDC$ (3 sides prop.)

Example of Student Work

(The notation $\angle C$ could cause confusion, but otherwise correct)

$\triangle ABC \sim \triangle EDC$

$\angle C = \angle C$ (對頂角)

$CD = CB \div 2$

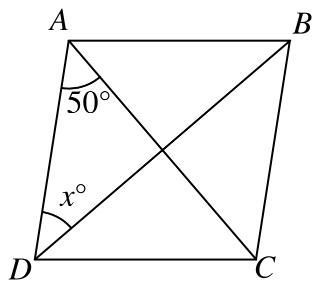
$CE = CA \div 2$

$\triangle ABC \sim \triangle EDE$ (二邊成比例而夾角相等)

- Pythagoras' Theorem: Most students could use Pythagoras' Theorem or its Converse to solve simple problems. However, they were confused about the differences between Pythagoras' Theorem and its Converse.

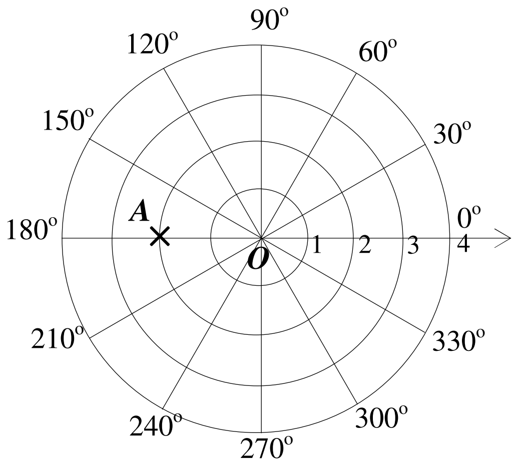
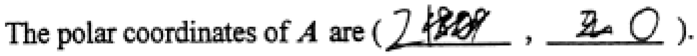
Q53/M2
Example of Student Work (Using Converse of Pythagoras' Theorem – could show improvement) 求 $BC^2 + DB^2 = DC^2$ $5^2 + 12^2 = 13^2$ $25 + 144 = 169$ ∴是直角三角形

- Quadrilaterals: Students performed satisfactorily. They could improve in dealing with items related to rhombuses and kites.

Q55/M3
Exemplar Item (rhombus) *In the figure, ABCD is a rhombus. Find the value of x.* 
Example of Student Work (wrongly assumed that base angles of rhombus are equal) $\angle D = 2x$ (菱形性質) $\angle A = \angle D$ (菱形性質) $50 \times 2 = 2x$ $100 = 2x$ $\frac{100}{2} = x$ $x = 50°$

Learning Geometry through an Analytic Approach

- Introduction to Coordinates: Students' performances were fair. In particular, performances were weak when they were asked to match a point under rotational transformation with its image.

Q43/M1
Exemplar Item (Polar coordinates) *Find the polar coordinates of point **A** in the figure.* 
Example of Student Work (confused with rectangular coordinates) 

- Coordinate Geometry of Straight Lines: Students' performances were fair. Some of them were confused with different conditions for parallel lines and perpendicular lines.

Trigonometry

- Trigonometric Ratios and Using Trigonometry: Most students showed certain degree of understanding in basic trigonometric ratios and applications. They showed improvement in items requiring them to solve right-angled triangles. However, they could improve in recognizing ideas in angle of depression and gradient.

Q52/M4
Example of Student Work (trigonometric ratio – solving correctly) 

S.3 Data Handling Dimension

S.3 students performance was fair in this Dimension. They did well in items requiring fewer working steps (such as interpret simple statistical charts, or finding median and mean from ungrouped data). However, performances were weak when students were asked to construct simple statistical charts. They also did not do well with items requiring deeper understanding (such as calculating theoretical probability, or distinguish discrete and continuous data). Comments on students' performances are provided below with examples cited where appropriate (question number *x* / sub-paper *y* quoted as Q*x*/M*y*). Other items worthy of attention may also be found in the section ***General Comments***.

Organization and Representation of Data

- Introduction to Various Stages of Statistics: Students' performances were fair. They could collect and organize data using simple methods. However, many were confused with discrete and continuous data.

Q56/M1

Example of Student Work (confused with tallies and frequencies)

表一

所需時間（分鐘）	頻數
1 – 10	///
11 – 20	/
21 – 30	////
31 – 40	////
41 – 50	//
51 – 60	////

表二

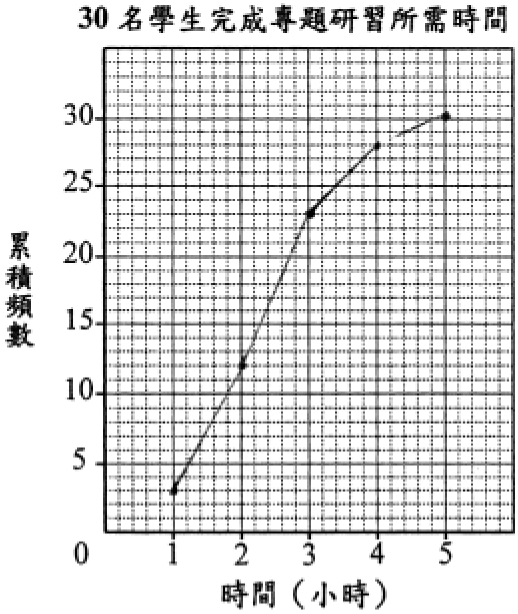
所需時間（分鐘）	頻數
1 – 15	////
16 – 30	////
31 – 45	卌
46 – 60	卌 //

- Construction and Interpretation of Simple Diagrams and Graphs: Students did quite well. Most students could interpret simple statistical charts. However, many of them could not draw cumulative frequency curves correctly. They also had difficulties identifying sources of deception in graphs.

Q54/M2

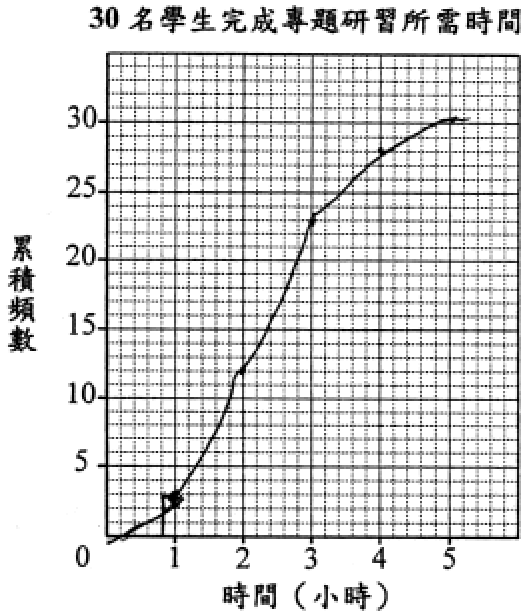
Example of Student Work

(drawing a cumulative frequency polygon instead of a curve)



Example of Student Work

(not able to draw a smooth cumulative frequency curve)



Analysis and Interpretation of data

- Measures of Central Tendency: Most students could find the median or mean from ungrouped data. However, they performed weakly when using grouped data.

Q46/M3
Exemplar Item (Using grouped data to find arithmetic mean) Less than half of the students answered this item correctly. *The following table shows the age distribution of 30 students:*

Age	11 – 13	14 – 16	17 – 19
Frequency	5	10	15

Find the mean age of these students.

Probability

- Simple Idea of Probability: Most students could compute empirical probability; however, their performance was weak when computing a theoretical probability by listing.

Q48/M1
Exemplar Item (calculate theoretical probability by listing) Only a few students answered this item correctly. *A fair $5 coin is tossed three times. Find the probability of getting exactly 2 Heads.*

General Comments on S.3 Student Performances

The overall performance of S.3 students was good. They did better in Number and Algebra Dimension. Performances were satisfactory in Measures, Shape and Space Dimension and Data Handling Dimension.

The areas in which students demonstrated adequate skills are listed below:

Directed Numbers and the Number Line:

- use positive numbers, negative numbers and zero to describe situations in daily life (e.g. Q21/M1)
- demonstrate recognition of the ordering of integers on the number line (e.g. Q22/M1)

Numerical Estimation:

- determine whether to estimate or to compute the exact value in a simple context (e.g. Q21/M4)

Approximation and Errors

- convert numbers in scientific notation to integers or decimals (e.g. Q1/M3)

Rational and Irrational Numbers

- demonstrate, without using calculators, recognition of the integral part of $\sqrt{a}$, where a is a positive integer not greater than 200 (e.g. Q1/M4)

Rate and Ratio

- demonstrate recognition of the difference between rate and ratio (e.g. Q24/M1)

Formulating Problems with Algebraic Language

- translate word phrases/contexts into algebraic languages (e.g. Q3/M2)
- substitute values into some common and simple formulas and find the value of a specified variable (e.g. Q25/M1)
- describe patterns by writing the next few terms in arithmetic sequences from several consecutive terms (e.g. Q26/M1)

Manipulations of Simple Polynomials

- multiply a monomial by a monomial (e.g. Q3/M4)

Formulas

- substitute values of formulas (in which all exponents are positive integers) and find the value of a specified variable (e.g. Q48b/M4)

Linear Inequalities in One Unknown

- use inequality signs $\geq$, $>$, $\leq$ and $<$ to compare numbers (e.g. Q35/M2)

Estimation in Measurement

- find the range of measures from a measurement of a given degree of accuracy (e.g. Q8/M2)

Introduction to Geometry

- demonstrate recognition of common terms in geometry such as regular polyhedra (e.g. Q11/M1)
- identify types of angles with respect to their sizes (e.g. Q12/M1)

Transformation and Symmetry

- determine the order of rotational symmetry from a figure and locate the centre of rotation (e.g. Q12/M3)
- demonstrate recognition of the effect on the size and shape of a figure under a single transformation (e.g. Q15/M1)

Angles related with Lines and Rectilinear Figures

- use the angle properties associated with intersecting lines/parallel lines to solve simple geometric problems (e.g. Q39/M1)

More about 3-D Figures

- name axes of rotational symmetries of cubes (e.g. Q15/M4)
- identify the nets of right prisms with equilateral triangles as bases (e.g. Q16/M1)

Pythagoras' Theorem

- use Pythagoras' Theorem to solve simple problems (e.g. Q53a/M2)

Introduction to Coordinates

- use an ordered pair to describe the position of a point in the rectangular coordinate plane and locate a point of given rectangular coordinates (e.g. Q41/M3)

Measures of Central Tendency

- find mean and median from a set of ungrouped data (e.g. Q46/M1, Q46/M2)

Simple Idea of Probability

- calculate the empirical probability (e.g. Q20/M2)

Other than items in which students performed well, the Assessment data also provided some entry points to strengthen teaching and learning. Items worthy of attention are discussed below:

Approximation and Errors

- Round off a number to a certain number of significant figures (e.g. Q2/M1): only about half of students chose the correct answer C. Some students treated the leftmost 0's as part of significant figures and chose A or B mistakenly.

Q2/M1
Round off 0.030 981 to 3 significant figures. A. 0.03 B. 0.031 C. 0.031 0 D. 0.030 98

Manipulations of Simple Polynomials

- Add or subtract polynomials: Two different items were set in the Assessment in different sub-papers. One item consisted of only one variable, whereas two different variables were in the polynomial of the other item.

Q28/M3
Simplify $(m - 2m^2) + (2m - 3m^2)$.

Q27/M4
Simplify $(x + 2y) - (3y - 2x)$.

The facility of Q27/M4 was clearly higher than that of Q28/M3. When answering Q28/M3, some students could not correctly handle terms with indices. For example:

Q28/M3
$(m - 2m^2) + (2m - 3m^2) =$ $2m^2 - 7m^3 + 6m^4$
$(m - 2m^2) + (2m - 3m^2) =$ $3m - 5m^4$

Laws of Integral Indices

- Use the laws of integral indices to simplify simple algebraic expressions (e.g. Q5/M2): some students forgot how to manipulate indices in the denominator and chose A mistakenly.

Q5/M2
Simplify $\frac{(c^2)^3}{c^{-3}}$ A. c^3 B. c^8 C. c^9 D. c^{11}

Linear Equations in Two Unknowns

- Determine whether a point lies on a straight line with a given equation (e.g. Q6/M2): About half of students chose the correct answer B. Many students did not match the values of x and y correctly and chose C.

Q6/M2
Which of the following is a point on the straight line $2y = x + 3$ *?* A. (–5 , –4) B. (–1 , 1) C. (2 , 1) D. (3 , 6)

- Plot graphs of linear equations in 2 unknowns: Two different items were set in the Assessment in different sub-papers. One of the items provided a table preset with some coordinates to assist plotting. The other item asked students to plot directly.

Q49/M2

Complete the following table for the equation $2y = x + 4$ *in the ANSWER BOOKLET:*

x	–4	0	4
y			4

Draw the graph of this equation on the rectangular coordinate plane given in the ANSWER BOOKLET.

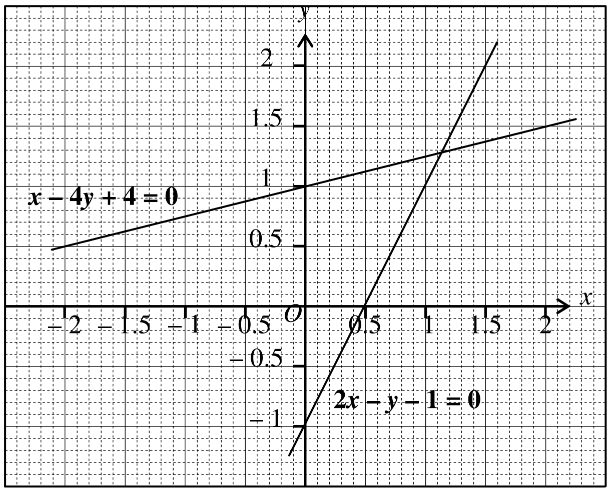
Q34/M1
Draw the graph of $x + y = 1$ *on the given rectangular coordinate plane in the ANSWER BOOKLET.*

Students did well in Q49/M2. However, the facility of Q34/M1 was only about half of that of Q49/M2.

- Be aware that the root obtained by the graphical method may not be exact (e.g. Q6/M1): Although more than half of students answered correctly (D), some students thought the root in the graph was an exact value (B).

Q6/M1

Solve graphically $\begin{cases} x-4y+4=0 \\ 2x-y-1=0 \end{cases}$



A. The exact solution is (1 , 1.5).
B. The exact solution is (1.1 , 1.3).
C. The approximate solution is (1 , 1.5).
D. The approximate solution is (1.1 , 1.3).

Linear Inequalities in One Unknown

- Demonstrate recognition of and apply the properties of inequalities (e.g. Q7/M3): Almost half of the students thought that C is the inequality needed according to the question.

Q7/M3

If $x > y$ *, which of the following is* ***INCORRECT****?*

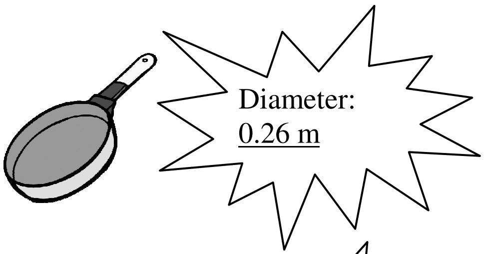
A. $2 + x > 2 + y$
B. $2 - x > 2 - y$
C. $2x > x + y$
D. $2x > 2y$

Estimation in Measurement

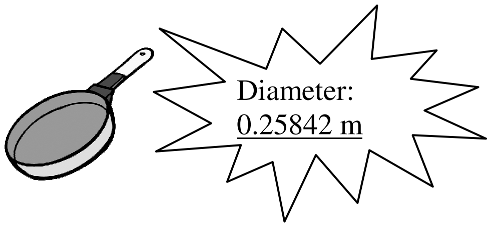
- Choose an appropriate unit and the degree of accuracy for real-life measurements (e.g. Q8/M4): Although many students answered correctly (C), some students thought that the advertisement with the most details was the most appropriate (D).

Q8/M4
The diameter of a frying pan is shown in each of the following advertisements. Which measurement is expressed in the most appropriate unit and degree of accuracy?

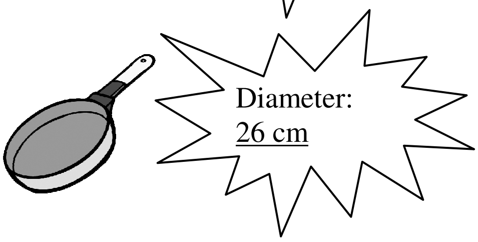
A.



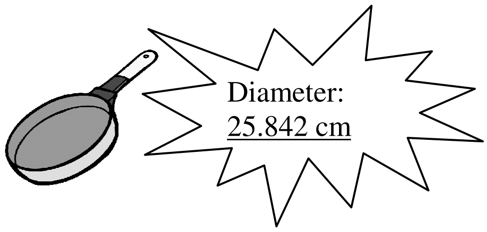
B.



C.



D.



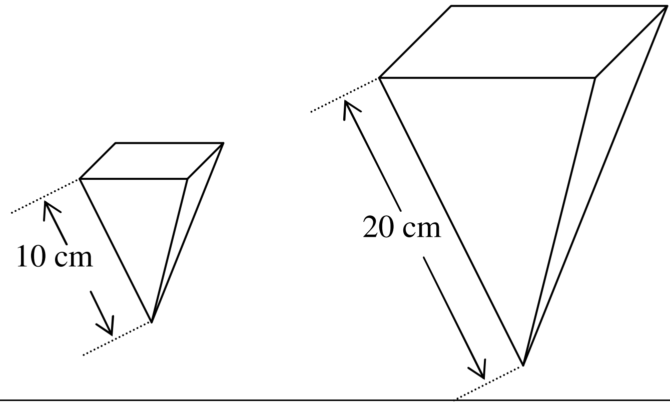
Simple Idea of Areas and Volumes

- Use the formulas for circumferences of circles (e.g. Q9/M2): Only half of students answered this item correctly (B). Some students chose the radius as the answer (A).

Q9/M2
The circumference of a circular table is 2π m. Find the diameter of the table. A. 1 m B. 2 m C. $\sqrt{2}$ m D. $2\sqrt{2}$ m

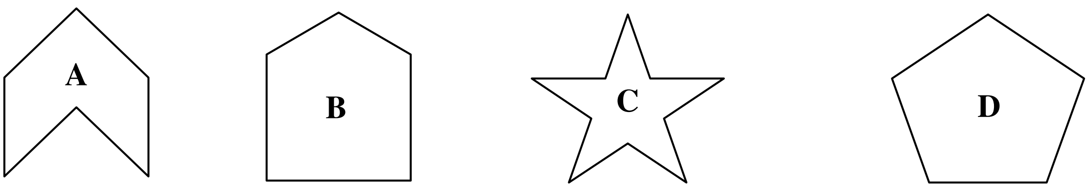
More about Areas and Volumes

- Use the relationships between sides and volumes of similar figures to solve related problems (e.g. Q10/M3): Almost half of students treated the relationship between sides the same as relationship between volumes (A). Only a few students answered correctly (D).

Q10/M3
In the figure, the lengths of corresponding slant edges of two similar pyramids are 10 cm and 20 cm respectively. If the volume of the small pyramid is V cm³, then the volume of the large pyramid is   A. $2V$ cm³.   B. $4V$ cm³.   C. $6V$ cm³.   D. $8V$ cm³.   

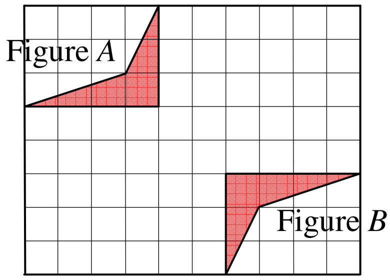
Introduction to Geometry

- Determine whether a polygon is convex (e.g. Q35/M4): Only a few students correctly pointed out that both B and D are convex polygons. Most students chose D as the only answer.

Q35/M4
Which of the following are convex polygons?   *(There may be more than one answer.)*   

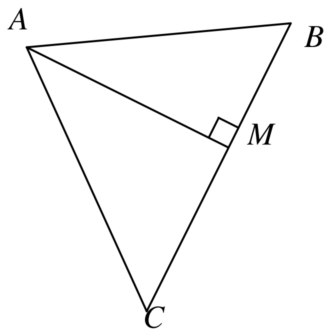
Transformation and Symmetry

- Name the single transformation involved in comparing the object and its image (e.g. Q13/M2): Less than half of students answered correctly (A). Almost half of students thought that reflectional transformation is involved (B).

Q13/M2	
Figure A is changed to Figure B after a single transformation. The transformation is A. a rotation. B. a reflection. C. a translation. D. an enlargement.	

Simple Introduction to Deductive Geometry

- Identify medians, perpendicular bisectors, altitudes and angle bisectors of a triangle (e.g. Q16/M3): Less than half of students chose the correct answer B. Some students thought that "perpendicular" was the only condition for "perpendicular bisector" and chose D mistakenly.

Q16/M3	
The figure shows ΔABC where $AM \perp BC$. AM must be A. a median of ΔABC . B. an altitude of ΔABC . C. an angle bisector of ΔABC . D. a perpendicular bisector of ΔABC .	

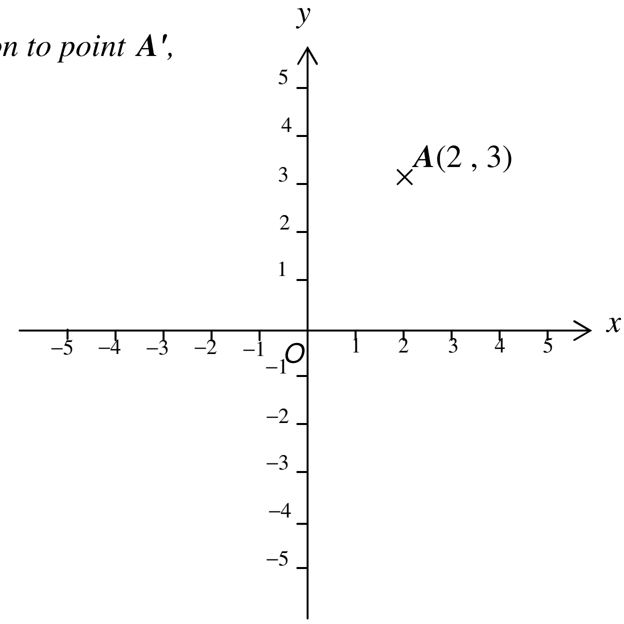
Introduction to Coordinates

- Match a point under a single transformation with its image in the rectangular coordinate plane (e.g. Q17/M4): Only a few students chose D correctly. Some students treated rotational transformation as reflectional transformation, and mistakenly chose C.

Q17/M4

If the point ***A***$(2\ ,\ 3)$ *is rotated* $90°$ *about the origin* O *in the anticlockwise direction to point* $\boldsymbol{A'}$*, then the coordinates of* $\boldsymbol{A'}$ *are*

A. $(2\ ,\ -3)$
B. $(3\ ,\ -2)$
C. $(-2\ ,\ 3)$
D. $(-3\ ,\ 2)$



Coordinate Geometry of Straight Lines

- Demonstrate recognition of the conditions for parallel lines and perpendicular lines (e.g. Q17/M3): About half of students chose B correctly. However, many students thought that L_2 and L_3 are perpendicular (C).

Q17/M3

The slopes of four straight lines L_1 , L_2 , L_3 *and* L_4 *are given in the following table:*

Line	L_1	L_2	L_3	L_4
Slope	5	-5	-5	$-\frac{1}{5}$

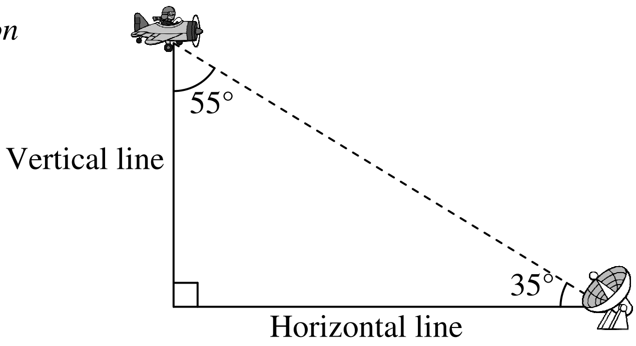
Which of the following pairs of straight lines are perpendicular to each other?

A. L_1 and L_2
B. L_1 and L_4
C. L_2 and L_3
D. L_3 and L_4

Trigonometric Ratios and Using Trigonometry

- Demonstrate recognition of the ideas of the angle of depression (e.g. Q18/M2): Almost half of students thought that the angle of depression was 55° (Correct answer was 35°).

Q18/M2
In the figure, the angle of depression of the radar from the plane is A. 35°. B. 55°. C. 90°. D. 125°.



Introduction to Various Stages of Statistics

- Distinguish discrete and continuous data (e.g. Q19/M1): Only about half of students chose "numbers of students" as discrete data correctly (B). Some students mistakenly chose "time records" (C).

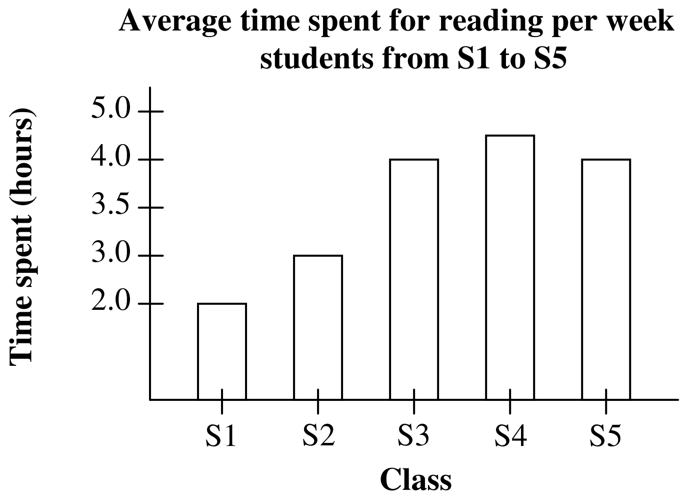
Q19/M1
Which of the following data is discrete? A. The heights of 30 students B. The numbers of students in 29 classes C. The time records of 8 runners D. The lengths of 10 cars

Construction and Interpretation of Simple diagrams and Graphs

- Identify sources of deception in misleading graphs (e.g. Q20/M4): Less than half of students chose B correctly. About the same number of students thought that the bar chart should have displayed the number of students at each level (C).

Q20/M4

The following graph shows the average time spent for reading per week of students from S1 to S5 of a secondary school:



Which of the following statements best explains why a reader could be misled by the graph?

A. The scale of horizontal axis is not consistent.
B. The scale of vertical axis is not consistent.
C. The number of students of each level is not shown in the graph.
D. The time spent (in hours) is not expressed in integers.

Performance of the Best S.3 Students in TSA 2008

Students were ranked according to their scores and the performances of the top 10% were analyzed further.

Most of these students either achieved the full maximum score or lost one or two score points in the Assessment. They demonstrated almost complete mastery of the concepts and skills assessed by the sub-papers attempted.

Q51/M1
Example of Student Work (Application of Ratio): could set up and solve the problem correctly with a complete solution. Let \$A be the amount in Renminbi $\frac{4000}{A} = \frac{100}{90}$ $A = 3600$ ∴ The amount in Renminbi is \$3600

Q55/M1
Example of Student Work (Geometric Proof): Understand clearly the requirement of the proposition in question, and use correct reasoning to set up the conclusion. AB = AD (Given) AC = AE (Given) ∠BAC = ∠DAE (Common) ∴ △ABC ≅ △ADE (S.A.S)

Q54/M3

Example of Student Work (Geometric Proof)

AB // CF 已知.

∵ AB // CF

∴ $\angle BCE = 65^\circ$ (內錯角 AB // CF)

∵ $\angle DEF = \angle BCE = 65^\circ$

∴ BC // DE (同位角相等)

Q49/M1

Example of Student Work (Estimation): Use estimation method relevant to the context of the problem.

Estimation of the total amount that a student has to pay for the textbooks = \$(150 + 150 + 85 + 85 + 70 + 70 + 70)

= \$680

The method is to round up.

Some common weak areas of high-achieving students are listed as follows:

- Some students could not draw cumulative frequency curve correctly.
- When the item did not provide a table to assist calculation of coordinates, some students could not plot graphs of linear equations.
- Some students could not correctly present the logical thinking needed by the use of Converse of Pythagoras' Theorem.
- Some students could not calculate theoretical probability.

Comparison of Students Performances in Mathematics at Secondary 3 TSA 2006, 2007 and 2008

TSA was conducted at S.3 for the third time in 2008. The percentages of students achieving Basic Competency from 2006 to 2008 are listed below:

Table 8.7 Percentages of S.3 Students Achieving Mathematics Basic Competency from 2006 to 2008

Year	% of Students Achieving Mathematics Basic Competency
2006	78.4
2007	79.9
2008	79.8

The percentage of S.3 students achieving mathematics basic competency in 2008 was about the same as last year.

The performances of S.3 students over the past three years in each Dimension are summarized below:

Number and Algebra Dimension

- Directed Numbers and the Number Line: Performance remained good.
- Numerical Estimation: There was continuous improvement on items requiring estimation and explanation. However, when the estimation result was given by the problem, most students still could not justify the reasonableness of the answers.
- Approximation and Errors: Performance declined on problems requiring conversion of significant figures.
- Rational and Irrational Numbers: Performance remained steady. There was room for improvement in usage of the number line.
- Using Percentages: Selling problems and compound-interest problems were still weak spots. However, this year students did better on simple-interest problems.
- Rate and Ratio: Performance remained steady. Students showed clear improvement in application of rate and ratio.
- Formulating Problems with Algebraic Language: Performance remained steady.

Students made slight improvements in some weak areas of past years, such as manipulation of sequences.

- Manipulations of Simple Polynomials: Students could distinguish polynomials from algebraic expressions better than past years. They also did better when the required operations involved multiplication by a monomial. However, students' performances were still weak in adding or subtracting polynomials and multiplication of binomials.
- Laws of Integral Indices: Performance remains fair. In particular, there was still room for improvement in simplifying algebraic expressions.
- Factorization of Simple Polynomials: Students made clear improvements in items using common factors or grouping terms, but there was still room for improvement. However, they regressed on items requiring the cross method.
- Linear Equations in One Unknown: Performance remained steady.
- Linear Equations in Two Unknowns: Performance remained steady. Most students needed help in order to plot graphs of linear equations. Moreover, when asked to determine whether a point lay on a straight line with a given equation, students' performance declined from past years.
- Identities: Performance remained fair. Students still didn't do well in items requiring difference of two squares and perfect square expressions.
- Formulas: Students made some improvement this year. They could deal with algebraic fractions better.
- Linear Inequalities in One Unknown: Performance remains fair. The weakness of students was still solving inequalities. They only made slight improvement this year.

Measures, Shape and Space Dimension

- Estimation in Measurement: Performances of students remained steady. They could better find the range of measures of a given degree of accuracy. They were better at estimation and giving explanations than past years. However, in items involving more judgments (such as reducing errors in measurements) performances clearly declined from past years.

- Simple Idea of Areas and Volumes: Performances of students remained steady. They continuously could apply formulas which were unique (such as volume of cylinder). However, students still did not do well in manipulations involving radius, diameter and circumference.

- More about Areas and Volumes: Students made slight improvements. Although performance was still weak in items dealing with relationships of sides and volumes in similar figures, they in general could make better use of formulas to calculate measures of various figures (such as surface area of cone).

- Introduction to Geometry: Students' performances varied with respect to different BC descriptors within this Unit. They made improvement in finding cross sections. However, performances were weaker when students were asked to identify types of polygons.

- Transformation and Symmetry: Performances of students remained good in general. It should be noted that they were sometimes confused with reflectional transformation and rotational transformation.

- Congruence and Similarity: Performance remained fair. When students didn't have to write out reasons themselves (such as multiple-choice questions), they continuously performed well. However, when the items required students to write out the reasons, they were still often confused with the conditions of congruence and of similarity.

- Angles related with Lines and Rectilinear Figures: Performances remained good. In general, students remained strong in solving geometric problems.

- More about 3-D Figures: Students continuously could deal with 3-D figures holistically (such as identifying the nets). However, performance was still weak when dealing with the angles, lines, and planes associated with 3-D figures.

- Simple Introduction to Deductive Geometry: Performances remained fair. As in past years, students were willing to try to write geometric proofs. However, they usually could not apply correct reasoning to complete the proofs.

- Pythagoras' Theorem: Performances remained steady. Using Pythagoras' Theorem to solve problems was as usual an area where students performed very well.

- Quadrilaterals: Performance regressed. Compared with past years, students did not do well in items involving rhombuses and kites.

- Introduction to Coordinates: Students performed well in general. However, they continuously regressed on items asking for matching of points under transformation with images.

- Coordinate Geometry of Straight Lines: Students performed well in general. When they had to use formulas, they still used the wrong formulas or manipulated carelessly from time to time.

- Trigonometric Ratios and Using Trigonometry: Performances remained steady in general. Students showed continuous improvement in solving right-angled triangles. However, they didn't do as well as in past years with items involving angle of depression and gradient.

Data Handling Dimension

- Introduction to Various Stages of Statistics: Students in general recognized various stages of statistics. However, distinguishing between discrete and continuous data remained weak. Moreover, this year they made significantly more mistakes when organizing data into groups.

- Construction and Interpretation of Simple Diagrams and Graphs: Except constructing statistical charts where students' performance was weak, they either performed well or showed improvement in other items.

- Measures of Central Tendency: Performances remained steady in general.

- Simple Idea of Probability: Performance regressed in items requiring calculation of theoretical probability by listing.

Comparison of Student Performances in Mathematics at Primary 3, Primary 6 and Secondary 3 TSA 2008

The percentages of P.3, P.6 and S.3 students achieving Basic Competency from 2004 to 2008 are as follows:

Table 8.8 Percentages of Students Achieving Mathematics Basic Competency

Year / Level	% of Students Achieving Mathematics BC				
	2004	**2005**	**2006**	**2007**	**2008**
P.3	84.9	86.8	86.9	86.9	86.9
P.6	--	83.0	83.8	83.8	84.1
S.3	--	--	78.4	79.9	79.8

A comparison of strengths and weaknesses of P.3, P.6, and S.3 students in TSA enables teachers to devise teaching strategies and tailor curriculum planning at different key stages to adapt to students' needs. The dimensions of Mathematics Curriculum at each key stage belong to different dimensions as shown below:

Table 8.9 Dimensions of Mathematics Curriculum for Primary 3, Primary 6 and Secondary 3

	Primary 3	**Primary 6**	**Secondary 3**
Dimension	Number	Number	Number and Algebra
		Algebra	
	Measures	Measures	Measures, Shape and Space
	Shape and Space	Shape and Space	
	Data Handling	Data Handling	Data Handling

The following table compares students' performances at P.3, P.6 and S.3 in Mathematics TSA 2008:

Table 8.10 Comparison of Student Performances in Mathematics at Primary 3, Primary 6 and Secondary 3 TSA 2008

Level / Dimension	P.3	P.6	S.3
Number	• Most P.3 students were capable of recognizing the place values of whole numbers but a small number of students confused the value of a digit with the place value. • Majority of the P.3 students did reasonably well in understanding the basic concepts and carrying out the arithmetic operations with numbers up to 3 digits. • Few P.3 students had forgotten the rule of "performing multiplication/division before addition/subtraction" when carrying out mixed operations. • Majority of the P.3 students did well in recognizing the relationship between fractions and the whole but had difficulty in understanding the concept of fractions as a part of one whole.	• Most P.6 students were capable of recognizing the place values of whole numbers. • The majority of the P.6 students did reasonably well in understanding the basic concepts and carrying out the arithmetic operations on whole numbers, fractions and decimals. • Some P.6 students had forgotten the rule of "performing multiplication/division before addition/subtraction" when carrying out mixed operations.	• Students understood directed numbers and their operations. • Students could operate the number line. • Most students could manipulate rate and ratio. • The use of scientific notation was not satisfactory.
	N. A.	• Many students were capable of choosing the appropriate mathematical expression to estimate the value of a given expression. However, some students could not apply estimation skills to solve more elaborate problems.	• Majority of students could do basic estimation. However, when they had to use their own words to explain the strategies, they didn't do as well.
	• Majority of the P.3 students could solve simple straightforward application problems and had showed a slight improvement in presenting their working steps when required but some students had difficulty in solving division problems where there was a remainder.	• The majority of the P.6 students could solve simple straightforward application problems and many of them could properly present their working steps. A considerable number of students had difficulty in solving application problems with more complicated or unfamiliar contexts.	• Students did well in using percentages to solve application problems which were more direct (such as finding simple interest). However, when the problem involved steps which were indirect (such as finding the cost), most students could not solve the problem satisfactorily.

Level / Dimension	P.3	P.6	S.3
Algebra	N. A.	• P.6 students were capable of using symbols to represent numbers. • They were capable of solving equations involving at most two steps in the solutions. • P.6 students were capable of solving problems by equations (involving at most two steps in the solutions).	• Students could use algebraic language to rewrite contexts. • Most students could solve simple equations. They could also substitute values into formulas to find the unknown value. • Most students knew the method of solving simultaneous equations. However, computation mistakes often intruded on their arriving at the correct solution. • When the variable term did not have any index, most students could carry out manipulations of simple polynomials. • Students were usually confused with variable terms which possessed indices. This happened in situations such as multiplication of polynomials and usage of integral index laws. They were weak in applying the correct index laws to simplify indices. A number of students could not identify simplified forms and incorrectly simplify further. • Some students did not understand concept of factorization of polynomials. This also caused them to incorrectly carry out procedures of factorization.

Level / Dimension	P.3	P.6	S.3
Measures	• Most P.3 students could identify and use Hong Kong Money. • P.3 students did well in exchanging but had difficulty when they were required to do simple calculations before exchanging money. • P3. students in general could indentify the start date/end date of the event.	• P.6 students were capable of applying the formula of circumference. • The majority of students could apply formulae to find the area of simple 2-D shapes. • P.6 students performed better than P.3 students in exchanging money. • P.6 students were outperformed by their P.3 counterparts in measuring the length of objects with 'ever-ready rulers' such as finger width.	• Some students were confused with the use of formula of circumference of circle. • Most students could use formulas of measures which were not easily confused (such as surface area, volume and arc length, etc).
	• Many P.3 students could measure the length of an object using millimetre or centimeter. • The majority of P.3 students were capable of measuring the weight of objects using grams and kilograms but declined slightly in measuring and comparing the weigh of objects using grams and kilograms.	• More P.6 students than P.3 students could measure the length of an object using millimetre or centimetre. • P.6 students performed better than P.3 students on measuring and comparing the weight of objects using grams and kilograms. • P.6 students performed better than P.3 students on choosing the appropriate unit of measurement for recording capacity.	• Most students could estimate measures. They could also give limited explanations of their estimations. • Students were weak in more abstract concepts (such as using relationship of similar figures to find measure, and the meaning of dimensions).

Level / Dimension	P.3	P.6	S.3
Shape and Space	• P.3 students were capable of identifying 2-D and 3-D shapes when these shapes are drawn in a commonly seen orientation but some students could not write the names of these shapes correctly. • P.3 students were capable of comparing the sizes of angles and recognizing right angles. • P.3 students in general could identify straight lines, curves, parallel lines and perpendicular lines. • Some P.3 students were unable to recognize the four directions when the given North direction was not pointing upwards.	• P.6 students could compare the size of angles and they did better than P.3 students in recognizing right angles. • P.6 students performed better than P.3 students in identifying 3-D shapes. • P.3 students were only required to recognize the four directions whereas P.6 students had to know the eight compass points. P.6 students performed significantly better than P.3 students in applying their knowledge of directions to solve problems.	• Students could manipulate figures of simple 3-D figure as a whole (such as the net of 3-D figure). • Most students could not satisfactorily deal with angles, line segments within 3-D figures. • Students had good knowledge of the rectangular coordinate system. However, they did only fairly when they had to further manipulate the coordinates (such as using the distance formula). • Students had good knowledge of plane geometric objects (such as angles, parallel lines, etc). • A strength of students is solving simple geometric problems. However, geometric proof remained the weakness of students. • Students could deal with simple symmetry and transformation. • Students sometimes confused the concept of congruence with similarity. • Using Pythagoras Theorem is a strength of students. However, they were weak in more conceptual applications (applying the Converse of Pythagoras Theorem).

Dimension \ Level	P.3	P.6	S.3
Data Handling	• P.3 students performed well at reading and interpreting simple pictograms with one-to-one representation. They were also good at answering simple questions based on such data, sometimes after carrying out simple calculations, but less so in answering open-ended questions. • P.3 students could construct statistical graphs from given data, though few of them did not draw their graphs clearly and neatly.	• P.6 students performed well at reading and interpreting pictograms and bar charts, including those of larger frequency counts. They were also good at answering simple questions involving simple calculations based on such data or information • P.6 students were relatively weak in making simple inferences/deductions from the data read from statistical graphs or answering questions based on further manipulation of such data. • P.6 students could construct statistical graphs from given data, though some of them did not draw their graphs clearly and neatly. • A small number of students made mistakes by inserting a number scale on the vertical axis of a pictogram to make it look like a "frequency axis" of a bar chart. Of special interest is that more students in P.6 than P.3 made such mistakes. • P.6 students were capable of solving simple problems of averages, but less capable of finding the average of a group of data.	• Students understood the basic procedures of statistical work. They could collect data using simple method. • Students could read and interpret simple statistical diagrams. • Most students could not draw statistical diagrams satisfactorily. • When dealing with misleading diagrams, students in general could not point out the misleading factor. • Students could calculate averages from ungrouped data. However, they did not do as well when using grouped data. • Students did better in dealing with misuse of averages. • Students did well when dealing with probability in situations similar to applying percentages. However, they did poorly when they had to calculate probability by listing.