Results of Secondary 3 Mathematics in TSA 2010

The territory-wide percentage of S.3 students achieving Mathematics Basic Competency in TSA 2010 was 80.1%. In 2009 the percentage was 80.0%.

Secondary 3 Assessment Design

The design of assessment tasks for S.3 was based on the documents *Mathematics Curriculum: Basic Competency for Key Stage 3 (Tryout Version)* and *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5), 1999.* The tasks covered the three dimensions of the mathematics curriculum, namely **Number and Algebra**, **Measures**, **Shape and Space**, and **Data Handling**. They focused on the Foundation Part of the S1 – 3 syllabuses in testing of the relevant concepts, knowledge, skills and applications.

The Assessment consisted of various item types including multiple-choice questions, fill in the blanks, answers-only questions and questions involving working steps. The item types varied according to the contexts of the questions. Some test items consisted of sub-items. Besides finding the correct answers, students were also tested in their ability to present solutions to problems. This included writing out the necessary statements, mathematical expressions and explanations.

The Assessment consisted of 165 test items (227 score points), covering all of the 129 Basic Competency Descriptors. These items were organized into four sub-papers, each 65 minutes in duration and covering all three Dimensions. Some items appeared in more than one sub-paper to act as inter-paper links. Each student was required to attempt one sub-paper only.

The composition of the sub-papers was as follows:

	Number of Items (Score Points)								
Sub-paper	Number and Algebra DimensionMeasures, Shape and Space Dimension		Data Handling Dimension	Total					
M1	23 (31)	22 (31)	6 (8)	51 (70)					
M2	25 (35)	22 (28)	5 (8)	52 (71)					
M3	23 (32)	21 (27)	7 (11)	51 (70)					
M4	24 (32)	20 (28)	7 (10)	51 (70)					
Total *	74 (98)	72 (101)	19 (28)	165 (227)					

 Table 8.5
 Composition of the Sub-papers

* Items that appeared in more than one sub-paper are counted only once.

The item types of the sub-papers were as follows:

Section	Percentage of Score Points	Item Types
А	~ 30%	• Multiple-choice questions: choose the best answer from among four options
В	~ 30%	Calculate numerical valuesGive brief answers
С	~ 40%	 Solve application problems showing working steps Draw diagrams or graphs Open-ended questions requiring reasons or explanations

 Table 8.6 Item Types of the Sub-papers

Performance of S.3 Students with Minimally Acceptable Levels of Basic Competence in TSA 2010

S.3 Number and Algebra Dimension

The performance of S.3 students was quite good in this Dimension. The majority of students demonstrated recognition of the basic concepts of directed numbers, linear equations in one unknown and 2 unknowns. Performance was only satisfactory in items related to Numerical Estimation and Formulas. Comments on students' performances are provided below with examples cited where appropriate (question number x / sub-paper y quoted as Qx/My). More examples may also be found in the section *General Comments*.

Number and Number Systems

- Directed Numbers and the Number Line: Most students could use directed numbers to describe situations in daily life. They could also handle the simple operation of directed numbers generally.
- Numerical Estimation: Many students could not determine whether to estimate or to compute the exact value in a simple context. Besides, when they were asked to estimate values with reasonable justifications, many students failed to justify their methods of estimation.

Q23/M1

Exemplar Item (determine whether to estimate or to compute the exact value)

In the following situations, are the values mentioned exact or estimated?

- (i) The capacity of High Island Reservoir is $273\,000\,000$ m³.
- (ii) The capacity of High Island Reservoir is 20 % more than that of Plover Cove Reservoir.

Q51/M1
Example of Student Work (find the exact value only)
兆明可購買 4. 份禮物。
原因:
他作買下三種禮腳包-份值, 解下 11.9元。
其頂色能再買一伤價值 \$ 9.8 南禮腳一伤,
熊下\$2.1。

Example of Student Work (without using the rounding-off method to estimate the price)

兆明可購買____ 份禮物。

原因:

回接每次 _____ 29.4+29.4 +9.8 7499 29+ 29+ 10 68 _____

Example of Student Work (good performance)

51. The number of gifts that can be bought by Terence is 5.

Reason :

Rounding up the puice to the hoavest \$10,
their prices are \$10, \$2 and \$30,
As Ference must buy at 180st 2 kinds of gifts with
different prices.
He Should Chouse the Cheapert 2, he can buy 5 gifts with
price \$10, and I gift with the price \$10 dollar.
1.e. 5 5×\$10 + \$20 = \$70

• Approximation and Errors: Almost half of the students were not able to determine that the number of zeros should be retained when rounding off a number to a certain number of significant figures. However, they did well in using scientific notation.

Q2/M2

Example of Student Work (Round off a number to 3 significant figures.) Round off 0.059 99 to 3 significant figures.

- A. 0.059 9
- B. 0.06
- C. 0.060
- D. 0.060 0
- Rational and Irrational Numbers: Most of the students could demonstrate recognition of the concept of irrational numbers. However, some students could not represent real numbers on the number line.

Comparing Quantities

Using Percentages: Students did well in solving simple problems on depreciations.
 Besides, they fared better on compound – interest problems than on simple – interest problems.

Q44/M2
Exemplar Item (solve problems on simple interest)
May deposited \$3 000 in a bank at a simple interest rate. After 3 years, she received the amount of \$3 270. Find (a) the interest received after three years; (b) the annual interest rate.
Example of Student Work (without express the annual interest rate in percentage)
a)3年後,共省的利息: 3270-3000 =\$270

• Rate and Ratio: Performance remained steady. They did well to solve problems related to rate. They could find the other quantity from a given ratio *a* : *b* and the value of either *a* or *b*. However, their performance was fair when they had to use ratio to solve simple real-life problems

Q44/M4
Exemplar Item (find the area of the football field by using ratio)
The ratio of the length of a football field to its width is 5 : 2. If the width is 40 m, find the area of the football field.
Example of Student Work (use ratio mistakenly to find the length of the football field)
設題是 χ : $\frac{\chi}{40} = \frac{2}{5}$
三人之之中 (1) 17
$= 64 \text{ D} ^2$
Example of Student Work (with a mistake in calculation of the area of the football field)
长度: 这长度为X 足球场面积:=
$\frac{1}{2} = \frac{1}{40} \qquad 40 \times 100$
$\frac{5}{2}x^{6} = \chi = 400 m^{2}$
100=X
Example of Student Work (correct solution)
設長度是 Q m
$\frac{5}{2} = \frac{4}{40}$
$\frac{200 = 2a}{a = 100}$
、 長度是」mm
面積
$=40 \times 100$
$= 4000 m^{2}$

Observing Patterns and Expressing Generality

• Formulating Problems with Algebraic Language: Students' performance was fair. Almost half of the students were not able to formulate simple inequalities from simple contexts. Many students could write down the next few terms in sequences

from several consecutive terms that were given. However, when they were asked to write the n^{th} term of the sequence, some of them gave the next term '729' following the given terms as the answer. Only some students could find the n^{th} term 3^n correctly.

Q24/M3

Exemplar Item (formulate simple inequalities from simple contexts)

Rhoda has A for transport expenses every month. Every day she goes to school by minibus and the fare is 3 each time. Rhoda takes the minibus *x* times this month and she does not spend all the money for transport by the end of the month.

Write down an inequality to represent the relationship between *x* and *A*.

Example of Student Work (could not determine which side has greater value)

不等式: <u>自日主 x (3)</u>

Example of Student Work (could not express the answer in inequality)

不等式: <u>A-(X×3)</u>

 Manipulations of Simple Polynomials: Students could do some basic manipulations with polynomials but they performed poorly in finding the degree of polynomials. Furthermore, almost half of the students could not distinguish polynomials from algebraic expressions.

Q26/M2

Exemplar Item (find the degree of the polynomial)

Find the degree of the polynomial $3x^4 - x^6 - 2x^5 - 10$.

Example of Student Work (take the sum of the indices of all terms to be the degree of polynomial)

多項式的次數是 _____15 ____。

• Laws of Integral Indices: Students in general could find the value of *aⁿ*, where *a* and *n* are integers. Some students' performance was fair when they had to simplify algebraic expressions using laws of integral indices.

Q45/M1

Example of Student Work (Using the laws of integral indices mistakenly)



Example of Student Work (Using the laws of integral indices mistakenly)



• Factorization of simple Polynomials: Most students demonstrated recognition of factorization as a reverse process of expansion. More than half of the students could factorize simple polynomials by taking out common factors correctly. When students were asked to apply cross method to factorize expressions of the form $ax^2 + bx + c$, the questions with a=1 and a=2 were given in different papers respectively. The performances of students in the former case were much better than in the latter.

Algebraic Relations and Functions

- Linear Equations in One Unknown: Students did well. Most of them could solve simple equations and also demonstrated understanding of the meaning of roots of equations.
- Linear Equations in Two Unknowns: Students' performance remained steady. Many students could use algebraic method and graphical method to solve linear simultaneous equations. They could formulate simultaneous equations from simple contexts. When the table was provided, almost half of the students could plot a graph of linear equation.

Q46/M1

Example of Student Work (Solving simultaneous equations – although students knew how to use the method of substitution, mistakes occurred in the computation)

(3xty=70-0)	
{ - y= 2x-30 - @	
把③ 准入 ①	Ex=8 H X2
3×1+(2x-30) = 70	y = 2(8) - 3c
5x -30 = 70	y = -14
tr = 40	V
x = }	1, x=8, y=-14
	U ·

Q47/M2

Example of Student Work

(drawing graph –without extending in two ends, a line segment was drawn instead)

$y = \frac{2-x}{2}$			
x	-2	0	2
У	2	1	ଚ

						J	?							
	T					1								
1														
T	-	1								1		-		<u> </u>
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			2-		1	$\frac{0}{1}$		-1			2		>3	
			2		1	0 1- 2- 3-		-1			2		<u>></u> 3	
			2		1	0 1- 2- 3-		_1			2		≯ 3	
			2		1	0 1 2- 3- 4-		-1			2		>3	x

Example of Student Work (good performance)

$$y = \frac{2-x}{2}$$

<u></u>			
x	-2	0	2
у	2	1	0

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	+	K		4				-
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• Identities: More than half of the students were not able to distinguish equations from identities. They fared better on using the difference of two squares than on perfect square expressions to expand simple algebraic expressions, but performance was only fair in general.

Q29/M2

Example of Student Work (Using difference of two squares in expansion – could not distinguish the difference between x^2 and 2x)

$$(x+2y)(x-2y) = 2x - 4y^{2y}$$

Q29/M4

Example of Student Work (wrongly to take $(a-b)^2 = a^2 - b^2$ as an identity)

$$(2x-5)^2 = - \frac{4x^2 - 25}{2}$$

Example of Student Work (using difference of two squares in expansion – used incorrect formula and did not understand the meaning of expansion)

$$(2x-5)^2 = (4 \times +25)(4 \times -27)$$

• Formulas: There was room for improvement in students' performances. They could find the value of a specified variable in the formula. However, when they were asked to simplify algebraic fractions and perform change of subject in simple formulas, their performance was unsatisfactory.

Q29/M1

Example of Student Work (simplify algebraic fractions – treated subtraction as multiplication)

$$\frac{3xy}{x^2} - \frac{3y}{2x} = \frac{9y^2}{2x^2}$$



 Linear Inequalities in One Unknown: Students' performance was more than satisfactory. They could use inequality signs to compare numbers, formulate linear inequalities in one unknown from simple contexts and represent inequalities on the number line. Nevertheless, only half of the students could solve simple linear inequalities in one unknown.

S.3 Measures, Shape and Space Dimension

S.3 students performed quite well in this Dimension. They could find measures in 2-D and 3-D figures, solve problems related to Transformation and Symmetry, Pythagoras' Theorem and simple application of trigonometry. However, more improvement could be shown in items related to 3-D figures and geometric proofs. Comments on students' performances are provided below with examples cited where appropriate (question number x /sub-paper y quoted as Qx/My). More items may also be found in the section *General Comments*.

Measures in 2-D and 3-D Figures

• Estimation in Measurement: Students did well. Many students could choose an appropriate unit and the degree of accuracy for real-life measurements, although some of them found difficulty in estimating measures with justification.

Q49/M4

Example of Student Work (estimate the area of the living room – only estimated the length and the width but no area showed)

月出行是4m 因度有41国地石单 底是7m 国存太生的有一个目生也不享

Example of Student Work (estimate the area of the living room – good performance)

客廳團+1m×4 ;展園+1度/1复: 大门约1m,窗户约2m,大门与于超过之词 距约1m、门窗闭距1m/窗至墙壁约1m~ 1.5m,1聽文度≈1m+1m+1m+2m+1.5m =75m。應面积全7.5m×4m=30m²

- Simple Idea of Areas and Volumes: Students' performance remained steady, only that some of them were negligent with the unit for the answer and had poor presentation in their work.
- More about Areas and Volumes: Students' performance was satisfactory. Quite a number of students could use formulas to calculate arc lengths, areas of sectors, measures of circular cones and spheres. However, they had difficulties in using relationships between sides and surface area of similar figures to solve problems. They were also not able to distinguish among formulas for lengths, areas, volumes by considering dimensions.

Q50/M4

Example of Student Work (calculate volume and curved surface area of circular cones – correct steps, but no units on the answers)

a) tr2h	b) Trl
$=\frac{1}{3}T(12)^{2}(5)$	$=\pi(12)(13)$
$= \frac{1}{3}\pi(144)(5)$	$= 156\pi$
= 1元 (720)	
= 240 TC	

Learning Geometry through an Intuitive Approach

• Introduction to Geometry: Students did well in problems relating to angles. Many of them could sketch the cross-section of a right cylinder and sketch a diagram of a cuboid, but they didn't demonstrate recognition of polygons.



- Transformation and Symmetry: Students' performance was good. They demonstrated recognition of basic concepts.
- Congruence and Similarity: Students' performance remained steady. They could state reasons in general, but they didn't demonstrate recognition of the conditions for congruent and similar triangles.

Q35/M2



Triangle A	Triangle B	Triangle C
16 34 18	16 ^{34°}	16 18

• Angles related with Lines and Rectilinear Figures: Students did quite well. Many of them could solve simple geometric problems by using the properties of angles and sides of triangles. However, their performance was satisfactory when they used the angle properties associated with intersecting lines/parallel lines to solve simple geometric problems.



• More about 3-D figures: More than half of the students could name planes of reflectional symmetries or axes of rotational symmetries of cubes according to context of item. Although some of students could identity the plane, they did not write down the name of plane in correct order (e.g. write *ABHE* instead of *ABEH*). They also did not do well when asked to name the angle between planes. Moreover, their performance was fair on items related to the nets of cubes and matching 3-D objects with various views.

Q32/M4

Q32/M4	
Exemplar Ite VABCD is ABCD. AB of intersection of AB. Name the and plane ABCD	em (name the angle between 2 planes) a right pyramid with a square base CD is a horizontal plane. E is the point on of AC and BD . M is the midpoint ngle between the plane VAB and the
Example of S	Student Work (could not write down the correct angle)
(1)	The angle between the plane VAB and the plane $ABCD$ is
(2)	平面 VAB 與平面 ABCD 的交角是 AB。
(3)	平面 VAB 與平面 ABCD 的交角是 VAD。

Learning Geometry through a Deductive Approach

• Simple Introduction to Deductive Geometry: More than half of the students could write some basic steps of a geometric proof, but many could not complete the proof correctly. Besides, about half of the students were able to identify medians of a triangle.



Example of Student Work (steps were partly correct, but the proof was not completed) 10(直象的都 Example of Student Work (correct proof) 1/1° 24(D = U • Q48/M1 Exemplar Item (Geometric proof)

Example of Student Work (the notation of the angles could cause confusion, and the reason for similar triangles was not given)

С

R

Q

S

K

··· ∠AKC = ∠BKD (對項角) ∠A = ∠B (歸角 PQ (/RS) ∠C = ∠D (歸旨 PQ //RS) ... AACK ≈ ABDK

In the figure, PQ // RS, line

segments AB and CD intersect

Prove that $\triangle ACK \sim$

at K.

 $\triangle BDK.$

Example of Student Work (without providing reason in the first line, and confusing "SSS" and "AAA")

ZADKR =CKBD Cat CILDB= CILCA

 Pythagoras' Theorem: Many students could use Pythagoras' Theorem to solve simple problems. However, their performance was satisfactory when they were asked to determine whether the given triangles were right-angled triangles or not by using the converse of Pythagoras' Theorem.



• Quadrilaterals: Students performed well. They could use the properties of trapeziums and rectangles in numerical calculations.

Learning Geometry through an Analytic Approach

• Introduction to Coordinates: Students' performance was fair. In particular, nearly half of the students could not match a point under reflection with respect to lines parallel to the *x*-axis with its image in the rectangular coordinate plane.



• Coordinate Geometry of Straight Lines: Students' performance was fair. Many students could use the formula to find the slope of the line. They demonstrated recognition of the conditions for parallel lines and perpendicular lines. However, only half of the students could use the distance formula to find the distance between 2 points.

Trigonometry

• Trigonometric Ratios and Using Trigonometry: Students showed certain degree of understanding in the sine, cosine and tangent ratios. They could find the angles when they were asked to solve right-angled triangles, but their performance was only satisfactory when solving the sides of the triangles. Besides, more than half of the students could solve simple problems related to gradient.

Q49/M1
Example of Student Work (solving the sides of a right-angled triangle – without
correcting the answer to 1 decimal place according to the requirement of the question,
and the variable <i>y</i> was undefined)
$\frac{\cos 340}{\cos 34-y} = \frac{y}{\cos 2}$
$h = HB > 665 = M_{H}$
Example of Student Work (good performance)
由D剩得C的俯角=由C剩得D的仰角(內始间)
$\cos 34^\circ = \overline{CD}$
$(0.534^\circ = \frac{AB}{800}$
$AB = S_{00}(\cos 34^{\circ})$
AB = 663.2
两個電車站角水平距翻呈663.2 m

S.3 Data Handling Dimension

The performance of S.3 students was fair in this Dimension. They showed recognition of various stages of Statistics. They also did well in items related to interpreting simple statistical charts and finding the arithmetic mean or mode from a set of ungrouped data. Many students could also calculate the empirical probability. However, performance was weak when students were asked to calculate the theoretical probability by listing and finding arithmetic mean from a set of grouped data. Comments on students' performance are provided below with examples cited where appropriate (question number x / sub-paper y quoted as Qx/My). More examples may also be found in the section *General Comments*.

Organization and Representation of Data

- Introduction to Various Stages of Statistics: Students' performance was quite good. Considerable number of students not only could collect and organize data using simple methods, but also distinguish discrete and continuous data.
- Construction and Interpretation of Simple Diagrams and Graphs: The performance of students was satisfactory. They didn't demonstrate recognition of the statistical charts. Most of them could not construct histograms correctly and chose scatter diagrams to present a set of data. On the other hand, most of the students did well in interpreting simple statistical charts.





Analysis and Interpretation of data

 Measures of Central Tendency: Most students could find the arithmetic mean or mode from ungrouped data. From a set of grouped data, they fared better on finding the median than on finding the arithmetic mean. Moreover, most of them could not identify sources of deception in cases of misuse of averages.

Q42/M4

Exemplar Item (find the arithmetic mean from a set of grouped data)

The table below shows the record of library books borrowed by students in December.

Number of books borrowed	1 – 5	6 – 10	11 – 15
Number of students	52	36	12

What is the mean number of books borrowed by each student?

Q51/M3

Q51/M3 Example of Student Work (without explained clearly why mode should not be used) (a) 偉傑的聲稱是把 作為平均值而得的。 (b) 我 * 同意 / 不同意 (*圈出正確答案) 偉傑的說法。 原因: 因為43%在每日最高相對温度是最低,應用算術平均數 不應用眾数。 95+183+78+62+56+43+43+45+50 Ŧ - 64.25 · 2009年年月最高相對路度是 64.259。 Example of Student Work (good performance) (a) Andy uses _ mode as the average in his claim. (b) I * agree / disagree (*circle the correct answer) with Andy's claim. Reason: 10 months in the year have the relative humidity more than 43%, only two months have the relative ... humidity of 43%. However, Andy said that the maximum ...monthly relative humidity is 43% it was totally wrong since most of the relative humidity is higher than 43%.

Probability

• Simple Idea of Probability: Most students could compute empirical probability, but their performance was weak when they were asked to calculate the theoretical probability by listing.

Q39/M2							
Exemplar Item (calculating theoretical probability by listing)							
Given that Mrs Tang has 3 children. Find the probability that Mrs Tang has only one boy.							
Example of Student Work (consider the number of children only)							
鄧太只有一名男孩的概率是。							
Example of Student Work (consider the sex of children only)							
鄧太只有一名男孩的概率是 56% 。							

General Comments on S.3 Student Performances

The overall performance of S.3 students was good. They did better in Number and Algebra Dimension and Measures, Shape and Space Dimension. Performance was fair in Data Handling Dimension.

The areas in which students demonstrated adequate skills are listed below:

Directed Numbers and the Number Line:

- Use positive numbers, negative numbers and zero to describe situations in daily life (e.g. Q21/M2).
- Demonstrate recognition of the ordering of integers on the number line (e.g. Q21/M3).

Approximation and Errors:

• Convert numbers in scientific notation to integers or decimals (e.g. Q2/M3).

Rational and Irrational Numbers

• Demonstrate recognition of the integral part of \sqrt{a} , where *a* is a positive integer not greater than 200 (e.g. Q2/M4).

Formulating Problems with Algebraic Language

• Describe patterns by writing the next few terms in sequences from several consecutive terms of integral values (e.g. Q25/M1).

Manipulations of Simple Polynomials

• Multiply a binomial by a monomial (e.g. Q6/M2).

Laws of Integral Indices

• Find the value of a^n , where a and n are integers (e.g. Q6/M4).

Factorization of Simple Polynomials

• Demonstrate recognition of factorization as a reverse process of expansion (e.g. Q7/M3).

Linear Equations in One Unknown

• Demonstrate understanding of the meaning of roots of equations (e.g. Q7/M4).

Linear Equations in Two Unknowns

• Formulate simultaneous equations from simple contexts (e.g. Q8/M3).

Linear Inequalities in One Unknown

• Use inequality signs \geq , >, \leq and < to compare numbers (e.g. Q29/M3).

Estimation in Measurement

• Choose an appropriate unit and the degree of accuracy for real-life measurements (e.g. Q10/M4).

Introduction to Geometry

• Make 3-D solids from given nets (e.g. Q13/M1).

Transformation and Symmetry

- Name the single transformation involved in comparing the object and its image (e.g. Q14/M2).
- Demonstrate recognition of the effect on the size and shape of a figure under a single transformation (e.g. Q14/M4).

Angles related with Lines and Rectilinear Figures

• Use the properties of angles of triangles to solve simple geometric problems (e.g. Q49/M2).

Introduction to Coordinates

• Use an ordered pair to describe the position of a point in the rectangular coordinate plane and locate a point of given rectangular coordinates (e.g. Q17/M2 and Q37/M3).

Construction and Interpretation of Simple Diagrams and Graphs

• Interpret simple statistical charts (e.g. Q42/M2).

Measures of Central Tendency

• Find mean, median and mode from a set of ungrouped data (e.g. Q42/M1 and Q41/M3).

Other than items in which students performed well, the Assessment data also provided some entry points to strengthen teaching and learning. Items worthy of attention are discussed below:

Manipulations of Simple Polynomials

• Distinguish polynomials from algebraic expressions (e.g. Q5/M2): The analysis showed that both abler and less able students didn't demonstrate recognition in the concept of polynomials. Less than half of the students chose the correct answer B. The numbers of students who chose other options (A, C or D) were almost the same.

Q5/1	M2
Whi	ch of the following is a polynomial?
A.	$\frac{x^2}{2y}$ - 3
B.	$\frac{x^2 - 2y}{3}$
C.	$x^2 - 2\sqrt{y}$
D.	$2^x - 2y$

• Demonstrate recognition of terminologies (e.g. Q4/M1): Half of the students chose the correct answer C. Some students thought that the polynomials had unlike terms only when the terms consisted of different variables. As a result, they chose A mistakenly.

Q4/M1	1
Which	of the following polynomials has / have unlike terms?
I.	5a + 5ab
II.	$4a^2 - 6a^2$
III.	$6a^2 + 6a$
A.	I only
B.	II only
C.	I and III only
D.	I, II and III

• Add or subtract polynomials of at most 4 terms (e.g. Q27/M4): If the variables in polynomials were more than one, the performance of students was weak and more than half of them could not find the correct answer.

Q27/M4
Simplify $(2a^2 + 3ab) - (a^2 - ab)$.
Example of Student Work
$(2a^2 + 3ab) - (a^2 + ab) = 3a^2 + 4ab$

• If a single variable polynomial was given and options were provided in the question (e.g. Q5/M1), most of the students could chose the correct answer.

Q5/M1

Simplify $5x^2 - 2x + 2x^2$. A. $2x^2 + 3x$ B. $7x^2 - 2x$ C. $5x^3$ D. $5x^2$

Linear Equations in Two Unknowns

• Plot graphs of linear equations in 2 unknowns: Two different items were set in the Assessment in different sub-papers. The equations in these two items were equivalent, only differing from the form. The result showed that the facility of Q47/M2 was higher than that of Q46/M3.

Q47/M2	2			
Comple	ete the table for th	e equation $y = \frac{2-x}{2}$	$\frac{x}{2}$ in the ANSW	ER BOOKLET.
	<i>x</i>	-2	0	2
	У		1	
Draw t ANSV	he graph of this VER BOOKLI	equation on the ET.	rectangular coord	linate plane given in the

Q46/M3

Complete the table for the equation x + 2y - 2 = 0 in the **ANSWER BOOKLET**.

x	-2	0	2
у		1	

Draw the graph of this equation on the rectangular coordinate plane given in the **ANSWER BOOKLET**.

Identities

• Tell whether an equality is an equation or an identity (e.g. Q8/M1): Less than half of the students chose the correct answer C, while almost same number of students chose B, showing that some of them took $(x+a)^2 = x^2 + a^2$ as an identity.

Q8/M1

Which of the following is an identity?

- A. 4(x-1) = 4x-1
- B. $(x+3)^2 = x^2 + 9$
- C. 4x + 2(x-1) = 2(3x-1)
- D. 7 3x = -(3x + 7)

Linear Inequalities in One Unknown

• Demonstrate recognition of the properties of inequalities (e.g. Q9/M4): Only half of the students chose the correct answer C. Quite a number of students thought that the inequalities in option A and B were wrong.

Q9/M4

If x > y, which of the following inequalities is **INCORRECT**? A. x + y > 2yB. 2 - x < 2 - yC. $\frac{x}{-2} > \frac{y}{-2}$ D. 2y < 2x More about Areas and Volumes

• Use the relationships between sides and surface areas/volumes of similar figures to solve related problems (e.g. Q20/M2): Some students treated the relationship between sides of two similar figures the same as the relationship between their surface areas and so they chose option A.



• Distinguish among formulas for lengths, areas and volumes by considering dimensions (e.g. Q31/M3): Considerable number of students thought that formula (i) represented the surface area of the right frustum, and formula (ii) represented the total sum of lengths.

Q31/M3

In the figure, the top and the base of the right frustum are squares of side lengths a and b respectively. The height of the frustum is h and the height of the lateral planes is s. By considering the dimensions, distinguish the following formulae according to the volume, the surface area, or the total sum of lengths of the frustum.



Introduction to Geometry

• Demonstrate recognition of common terms in geometry (e.g. Q11/M2): Students in general could identify the relationship between 'regular polygons' and 'equilateral', but they were not aware whether all interior angles were equal or not. Half of the students chose the correct answer D, while some students thought that any rhombus must be a regular polygon.

Q11/M2

Which of the following descriptions of polygons **MUST** be correct?

- A. Any rhombus must be a regular polygon.
- B. Any isosceles triangle must be a regular polygon.
- C. All interior angles of any regular polygon must be acute.
- D. All sides of any regular polygon must be equal in length.
- Determine whether a polygon is regular, convex, concave, equilateral or equiangular (e.g. Q32/M3): Most students thought that the polygons in figure A and E were equiangular. A few students could identify that only figure C and figure D were correct answers.



Congruence and Similarity

Demonstrate recognition of the conditions for congruent and similar triangles (e.g. Q14/M1): Some students were confused with the conditions for congruent and similar triangles. They thought that 'AAA' was one of the conditions for congruent triangles.



Simple Introduction to Deductive Geometry

• Identify medians, perpendicular bisectors, altitudes and angle bisectors of a triangle (e.g. Q17/M3): Some students were confused with medians and altitudes.



Coordinate Geometry of Straight Lines

• Use the mid-point formula (e.g. Q18/M2): For applying the mid-point formula, some students took the formula as $(x_1 + x_2, y_1 + y_2)$ or $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$ mistakenly.

018	/M2
VI0	11112

A(-6, 8) and B(4, -2) are two points in the rectangular coordinate plane. The midpoint of AB is

A. (-1, 3). B. (-2, 6). C. (-5, 5). D. (-10, 10).

Construction and Interpretation of Simple Diagrams and Graphs

• Choose appropriate diagrams/graphs to present a set of data (e.g. Q16/M2): Only half of the students knew to choose scatter diagram to find out whether 2 sets of data relate to each other. Another half of the students chose cumulative frequency polygon or histogram.

Q16/M2															
The table below shows the marks of 15 students in English and Music tests.															
Students	Α	B	С	D	Ε	F	G	Η	Ι	J	K	L	Μ	Ν	0
English test	12	67	33	74	86	24	47	90	73	23	64	42	83	49	65
Music test	16	55	38	81	79	20	42	86	68	24	68	51	86	46	71
Mr Ho wants to use a statistical graph to find out whether the marks of the 2 tests															

relate to each other. Which of the following graphs is the most suitable?

- A. Scatter diagram
- B. Cumulative frequency polygon
- C. Pie chart
- D. Histogram

Read information (including percentiles, quartiles, median) and frequencies from diagrams/graphs (e.g. Q20/M4): Only half of the students could choose the correct answer B. Some of them took the number of students who failed the test to be the answer (40 × 60% = 24).



Best performance of S.3 Students in TSA 2010

Students were ranked according to their scores and the performances of the top 10% were analyzed further.

Most of these students either achieved the full maximum score or lost one or two score points in the Assessment. They demonstrated almost complete mastery of the concepts and skills assessed by the sub-papers attempted.

Most of these students were able to add, subtract and multiply polynomials (e.g. Q5/M1 and Q6/M2), find the value of a^n , where a and n are integers (e.g. Q6/M4), solve simple problems on compound interest, compounded yearly (e.g. Q43/M3), use rate and ratio to solve simple real-life problems (e.g. Q45/M3), demonstrate recognition of factorization as a reverse process of expansion (e.g. Q7/M3), use the formulas for circumferences and areas of circles (e.g. Q47/M1), identify the image of a figure after a single transformation (e.g. Q13/M3), use the properties of angles of triangles to solve simple geometric problems (e.g. Q49/M2 and Q49/M3), use the properties of rectangles in numerical calculations (e.g. Q50/M3), find mode from a set of ungrouped data (e.g. Q41/M3) and construct simple statistical charts (e.g. Q50/M1).

The examples of work by these students are illustrated below:

Students with the best performance could set up and solve the problem correctly with a complete solution.

Q44/M2	
Example of Student Work (solve problems on simple interest)	
(a) The interest	
=\$3270-\$3000	
= \$270//	
(b) Let R be the annual interest rate.	
$$3000 \times R \times 3 = 270	
R = 0.03	
: The annual interest rate is 3% .	

Q47/M4
Example of Student Work (solve simple selling problems)
Let the marked price be $\frac{1}{20\%} - \frac{300}{200} = \frac{100}{200}$ $08_{y} = 500$ $y = 500 \div 0.8$ y = 625
The marked price of the mobile phone is \$625.

Students with the best performance could construct simple statistical charts by the given data.



Students with the best performance could make good use of the given conditions and solve the problem systematically.

Q49/M3

Example of Student Work (use the relations between sides and angles associated with isosceles/equilateral triangles to solve simple geometric problems)



Students with the best performance could show steps clearly and used correct reasoning to set up the conclusion.

Q50/M2
Example of Student Work (geometric proof)
AB = AC (BE)
BD = CD (2 p)
$AD = AD (\dot{u} \\ \ddot{x} \\ \ddot{a})$
$\therefore \ \Delta ABD \cong P ACD (S.S.S)$

Some common weak areas of high-achieving students are listed as follows:

- Some students could not determine whether to estimate or to compute the exact value in a simple context.
- Some students could not estimate values with reasonable justifications.
- Some students could not distinguish polynomials from algebraic expressions.
- Some students could not calculate the theoretical probability by listing.
- Many students could not determine whether a polygon is equiangular.
- Many students could not identify sources of deception in cases of misuse of averages.

Comparison of Student Performances in Mathematics at Secondary 3 TSA 2008, 2009 and 2010

This was the fifth year that Secondary 3 students participated in the Territory-wide System Assessment. The percentage of students achieving Basic Competency in this year was 80.1% which was about the same as last year.

The percentages of students achieving Basic Competency from 2008 to 2010 are listed below:

Year	% of Students Achieving Mathematics Basic Competency
2008	79.8
2009	80.0
2010	80.1

Table 8.7Percentages of S.3 Students Achieving Mathematics Basic
Competency from 2008 to 2010

The performances of S.3 students over the past three years in each Dimension of Mathematics are summarized below:

Number and Algebra Dimension

- Directed Numbers and the Number Line: The performance of students remained good over the past three years.
- Numerical Estimation: There was room for improvement of students' performance. Performance declined slightly when students were asked to estimate values with reasonable justifications.
- Approximation and Errors: Performance was steady on conversion of significant figures and convert numbers in scientific notation to integers.
- Rational and Irrational Numbers: Performance remained steady. In the usage of number line, the performance of students was not as good as past years.
- Using Percentages: Performance was still weak in the presentation of the answers: could not master the concept of percentage (e.g. confused with 3 and 3%), unit omitted, presentation unclear and incomplete.
- Rate and Ratio: Performance remained steady. Negligence of the unit still

happened in solving simple real-life problems.

- Formulating Problems with Algebraic Language: The performance of students declined in translating word phrases/contexts into algebraic languages. They did better in writing the next few terms in sequences from several consecutive terms. Solving the problems involving the n^{th} term of number sequence were still the weak spots.
- Manipulations of Simple Polynomials: There was room for improvement in distinguishing polynomials from algebraic expressions. However, students did better in addition or subtraction of polynomials.
- Laws of Integral Indices: Performance declined slightly in the problems involving negative indices.
- Factorization of Simple Polynomials: Students performed better in factorizing simple polynomials by taking out common factors or grouping terms. Performance remained satisfactory in factorization of simple polynomials by using the difference of two squares, the perfect square expressions or the cross method.
- Linear Equations in One Unknown: Students regressed on items related to solve simple equations, but they showed significant improvement in understanding of the meaning of roots of equations.
- Linear Equations in Two Unknowns: Performance remained steady in plotting graphs of linear equations. Performance declined in showing recognition that graphs of equations of the form ax + by + c = 0 are straight lines. Nevertheless, they were more able to formulate simultaneous equations from simple contexts.
- Identities: Performance of students was similar to last year when they were asked to expand simple algebraic expressions by using the difference of two squares and the perfect square expressions.
- Formulas: Performance was still weak in performing a change of subject in simple formulas but not including radical sign.
- Linear Inequalities in One Unknown: Students made improvement when using inequality signs to compare numbers and solving inequalities.

Measures, Shape and Space Dimension

- Estimation in Measurement: Performance of students was steady. They performed well in choosing an appropriate unit and the degree of accuracy for real-life measurements, but declined in choosing an appropriate measuring tool and technique. Performance was similar to last year in items relating to estimate measures with justification and reduce errors in measurements.
- Simple Idea of Areas and Volumes: Students performed better in calculation involving radius, circumference and area of circle. They made improvement in applying formulas for surface areas.
- More about Areas and Volumes: Same as past 2 years, students were able to calculate the measures of various figures (e.g. areas of sectors and volumes of spheres, etc) in general. However, performances remained unsatisfactory in items dealing with relationships of sides and surface areas in similar figures and distinguishing among formulas for lengths, areas, volumes by considering dimensions.
- Introduction to Geometry: Students did better in sketching simple solids and their cross-sections. They performed well when they were required to make 3-D solids from given nets. However, they were still weak in recognition of polygons.
- Transformation and Symmetry: Performances of students were quite good in general, although they regressed slightly on the problems involving rotation.
- Congruence and Similarity: Students could use the properties of congruent and similar triangles to solve triangles. However, they were confused with the conditions for congruent and similar triangles.
- Angles related with Lines and Rectilinear Figures: Students could use the properties of triangles to solve simple geometric problems. On the other hand, their performance was still fair when using the formulas for the sums of the interior angles and exterior angles of convex polygons.
- More about 3-D Figures: Performance was fair. They regressed slightly in matching 3-D objects from 2-D representations. Performance was still weak when dealing with the angles, lines, and planes associated with 3-D figures.
- Simple Introduction to Deductive Geometry: As in past years, students were unfamiliar with some theorems. For instance, they mixed up the meaning of

(alt \angle s eq.) and (alt \angle s, AB//CD). Therefore, they usually could not give correct reasons to perform simple proofs.

- Pythagoras' Theorem: Performances remained steady. Students were capable to apply Pythagoras' Theorem to solve simple problems.
- Quadrilaterals: Performance remained good. Students remained strong in solving geometric problems in general.
- Introduction to Coordinates: Students' performance remained steady. They demonstrated good recognition of the coordinates system. However, they were still weak in items relating to calculate areas of simple figures and transformation.
- Coordinate Geometry of Straight Lines: Performance declined in using the formula related to slope. However, they showed improvement on the application of mid-point formula.
- Trigonometric Ratios and Using Trigonometry: Performances remained steady in general. Students showed continuous improvement on the recognition of trigonometric ratios. Performance declined slightly in solving simple 2-D problems involving one right-angled triangle.

Data Handling Dimension

- Introduction to Various Stages of Statistics: Students showed significant improvement in distinguishing between discrete and continuous data. Besides, their performance was fair in problems related to organize the same set of data by different grouping methods and use simple methods to collect data.
- Construction and Interpretation of Simple Diagrams and Graphs: Performance was steady in interpreting and constructing simple statistical charts. However, they didn't do well in choosing appropriate diagrams, comparing the presentations of the same set of data by using statistical charts and identifying sources of deception.
- Measures of Central Tendency: Except the item related to identify sources of deception in misleading graphs/accompanying statements, students either performed steady or showed improvement in other items.
- Simple Idea of Probability: Students' performance varied in these 2 BC. Their performance remained good in calculating the empirical probability. However, they were not as strong in calculating the theoretical probability by listing.

Comparison of Student Performances in Mathematics at Primary 3, Primary 6 and Secondary 3 TSA 2010

The percentages of P.3, P.6 and S.3 students achieving Basic Competency from 2004 to 2010 are as follows:

Year	% of Students Achieving Mathematics BC						
Level	2004	2005	2006	2007	2008	2009	2010
P.3	84.9	86.8	86.9	86.9	86.9	#	87.0
P.6		83.0	83.8	83.8	84.1	#	84.2
S.3			78.4	79.9	79.8	80.0	80.1

 Table 8.8
 Percentages of Students Achieving Mathematics Basic Competency

Due to Human Swine Influenza causing the suspension of primary schools, the TSA was cancelled and no data has been provided.

A comparison of strengths and weaknesses of P.3, P.6, and S.3 students in TSA enables teachers to devise teaching strategies and tailor curriculum planning at different key stages to adapt to students' needs. The dimensions of Mathematics Curriculum at each key stage belong to different dimensions as shown below:

Table 8.9Dimensions of Mathematics Curriculum for Primary 3, Primary 6 and
Secondary 3

	Primary 3	Primary 6	Secondary 3	
Dimension	Numbor	Number	Number and Alcohre	
	INUIIDEI	Algebra	Number and Argeora	
	Measures	Measures	Measures, Shape and	
	Shape and Space	Shape and Space	Space	
	Data Handling	Data Handling	Data Handling	

The following table compares students' performances at P.3, P.6 and S.3 in Mathematics TSA 2010:

Level Dimension	P.3	P.6	S.3
Number	• Most P.3 students were capable of recognising the place values in whole numbers.	• P.6 students were capable of recognising the place values in whole numbers and decimals.	• The majority of students understood directed numbers and their operations.
	 Majority of the P.3 students could perform arithmetic calculations with numbers up to 3 digits. Some P.3 students forgot the computational rule of "performing multiplication/division before addition/subtraction" when carrying out mixed operations. P.3 students could compare fractions and recognise the relationship between fractions and the whole. Some students could not fully understand the basic concept of a fraction as parts of one whole. 	 The majority of the P.6 students could perform arithmetic operations on whole numbers, fractions and decimals. Some P.6 students forgot the computational rule of "performing multiplication/division before addition/subtraction" when carrying out mixed operations. P.6 students could compare fractions and understand the concept of a fraction as parts of one whole 	 Students could operate the number line. Many students did well in using scientific notation. Students fared better on the manipulation of rate to solve the problems than on the manipulation of ratio.
	N. A.	• Many students were capable of choosing the appropriate mathematical expression to estimate the value of a given expression.	• Majority of students were capable of choosing a reasonable expression to estimate answers from computations. However, when they explained the strategies, their performance was fair.
	• Majority of the P.3 students could solve simple application problems by presenting clear working steps and explanations. Students' performance was weaker in managing problems involving division in the calculation of money.	• The majority of the P.6 students could solve application problems by presenting clear working steps and explanations. Some students had difficulty in solving application problems with more complicated or unfamiliar contexts.	• Students did well in using percentages to solve simple application problems (such as selling problems). However, when the problem involved more parts and working steps (such as finding annual interest rate), most students could not present their steps correctly.

Table 8.10 Comparison of Student Performances in Mathematics at Primary 3, Primary 6 and Secondary 3 TSA 2010

Level Dimension	P.3	P.6	S.3
Algebra		• P.6 students were capable of using symbols to represent numbers.	• Students could formulate linear equations or linear inequalities from simple contexts.
		 They were capable of solving equations involving at most two steps in the solutions. P.6 students were capable of solving problems by simple equations. 	 Most students could solve simple equations. They could also substitute values into formulas to find the unknown value. Most students could multiply a monomial by a
	N. A.		 monomial or a binomial by a monomial. Students' performance was fair when they had to simplify algebraic expressions with negative indices.
			 Students didn't demonstrate recognition of and apply the properties of inequalities. Students' performance was fair in factorization and expansion of simple polynomials.

Level	P.3	P.6	S.3
Measures	 Many P.3 students could identify and use Hong Kong Money. P.3 students performed well in exchanging but had difficulty when they were required to do simple calculations before exchanging money. P.3 students had difficulty in indentifying the start date/end date of an event. 	 P.6 students could measure length or distance with 'ever-ready rulers'. P.6 students were capable of applying the formula of circumference. The majority of students could measure or calculate the perimeters and areas of simple 2-D shapes. The majority of students could find the volume of cubes and cuboids but was weak in understanding the relationship between capacity and volume. P.6 students could solve simple problems involving speed. 	 Students could choose an appropriate unit and the degree of accuracy for real-life measurements. Students could use the formulas for circumferences and areas of circles. Most students could use the formulas for surface areas and volumes of simple solids.
	 Many P.3 students could measure the length of an object using millimetre or centimetre. P.3 students were capable of measuring the weight of objects using grams and kilograms but weak in measuring and comparing the weight of objects. 	 P.6 students performed better than P.3 students on measuring the length of objects and the distance between objects. P.6 students performed better than P.3 students on measuring and comparing the weight of objects using grams and kilograms. P.6 students performed better than P.3 students on choosing the appropriate unit of measurement for recording capacity. 	 Most students could estimate measures. They could also give simple explanations of their estimations. Students were weak in more abstract concepts (such as using relationship of similar figures to find measures, and the meaning of dimensions).

Level	P.3	P.6	S.3
Shape and Space	 P.3 students were capable of identifying 2-D and 3-D shapes when these shapes are drawn in a commonly seen orientation. P.3 students did well in comparing the sizes of angles and recognizing right angles. P.3 students could identify straight lines, curves, parallel lines and perpendicular lines. But less capable in identifying parallel lines in 2-D shapes. P.3 students performed well in recognizing the four directions. 	 P.6 students did better than P.3 students in identifying parallel lines and perpendicular lines. P.6 students performed better than P.3 students in identifying 3-D shapes. P.3 students were only required to recognize the four directions whereas P.6 students had to know the eight compass points. P.6 students performed better than P.3 students in applying their knowledge of directions to solve problems. 	 Students could use common notations to represent angles and identify types of angles with respect to their sizes. Students could identify the relation between simple 3-D solids and their corresponding 2-D figures. They could also sketch cross-sections of simple solids. Most students could not satisfactorily deal with the angle between a line and a plane and the angle between 2 planes. Students could deal with simple symmetry and transformation. Students sometimes were confused the concept of congruence with similarity. Students had good knowledge of the rectangular coordinate system. However, they did only fairly when they had to manipulate problems involving transformation or using the distance formula. Students performed well in numerical calculations of simple geometric problems. However, geometric proof remained the weakness of students. Students could use Pythagoras Theorem to solve problems. However, their performance was fair when applying the Converse of Pythagoras Theorem.

Level	P.3	P.6	S.3
Data Handling	 P.3 students performed well at reading simple pictograms with one-to-one representation. They could answer straightforward questions by retrieving data but were weak in making deductions from the information. P.3 students could construct pictograms using one-to-one representation. Some of them could not give a suitable title and unnecessarily added a 'frequency axis' to a pictogram. 	 P.6 students performed well at reading pictograms and bar charts, including those of greater frequency counts. They could answer questions by interpreting the information. P.6 students could construct statistical graphs from given data, though some of them did not draw their graphs accurately. A small number of P.6 students mistakenly added a 'frequency axis' to a pictogram. P.6 students were capable of finding the average of a group of data and solving simple problems of averages. 	 Students understood the basic procedures of statistical work. They could collect data using simple method. Students could distinguish discrete and continuous data. Students could read and interpret simple statistical diagrams. Most students could not draw statistical diagrams satisfactorily. When dealing with misleading diagrams, students in general could point out the sources of deception. However, most of the students could not explain the reason of deception by their own words in cases of misuse of averages. Students did better in dealing with misuse of averages. Students did well in calculating the empirical probability. However, they did poorly when they had to calculate probability by listing.