Results of Secondary 3 Mathematics in TSA 2011

The territory-wide percentage of S.3 students achieving Mathematics Basic Competency in TSA 2011 was 80.1%. The proportion achieving basic competency in 2011 was almost the same as that of last year.

Secondary 3 Assessment Design

The design of assessment tasks for S.3 was based on the documents *Mathematics Curriculum: Basic Competency for Key Stage 3 (Tryout Version)* and *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5), 1999.* The tasks covered the three dimensions of the mathematics curriculum, namely **Number and Algebra**, **Measures**, **Shape and Space**, and **Data Handling**. They focused on the Foundation Part of the S1 – 3 syllabuses in testing the relevant concepts, knowledge, skills and applications.

The Assessment consisted of various item types including multiple-choice questions, fill in the blanks, answers-only questions and questions involving working steps. The item types varied according to the contexts of the questions. Some test items consisted of sub-items. Besides finding the correct answers, students were also tested in their ability to present solutions to problems. This included writing out the necessary statements, mathematical expressions and explanations.

The Assessment consisted of 165 test items (218 score points), covering all of the 129 Basic Competency Descriptors. These items were organized into four sub-papers, each 65 minutes in duration and covering all three Dimensions. Some items appeared in more than one sub-paper to act as inter-paper links. Each student was required to attempt one sub-paper only.

The composition of the sub-papers was as follows:

Sub-paper	Number of Items (Score Points)			
	Number and Algebra Dimension	Measures, Shape and Space Dimension	Data Handling Dimension	Total
M1	23 (30)	22 (30)	7 (11)	52 (71)
M2	25 (35)	21 (27)	6 (9)	52 (71)
M3	25 (36)	21 (28)	6 (7)	52 (71)
M4	25 (36)	21 (26)	6 (8)	52 (70)
Total *	76 (101)	71 (93)	18 (24)	165 (218)

 Table 8.5
 Composition of the Sub-papers

* Items that appeared in more than one sub-paper are counted only once.

The item types of the sub-papers were as follows:

Section	Percentage of Score Points	Item Types	
А	~ 30%	• Multiple-choice questions: choose the best answer from among four options	
В	~ 30%	Calculate numerical valuesGive brief answers	
С	~ 40%	 Solve application problems showing working steps Draw diagrams or graphs Open-ended questions requiring reasons or explanations 	

 Table 8.6 Item Types of the Sub-papers

Performance of S.3 Students with Minimally Acceptable Levels of Basic Competence in TSA 2011

S.3 Number and Algebra Dimension

The performance of S.3 students was steady in this Dimension. The majority of students demonstrated recognition of the basic concepts of directed numbers, rate and ratio, laws of integral indices and linear equations in 2 unknowns. Performance was only satisfactory in items related to Numerical Estimation. Comments on students' performances are provided below with examples cited where appropriate (question number x / sub-paper y quoted as Qx/My). More examples may also be found in the section *General Comments*.

Number and Number Systems

- Directed Numbers and the Number Line: Most students could demonstrate recognition of the ordering of integers on the number line. They could also handle the simple operation of directed numbers.
- Numerical Estimation: Many students were able to determine whether to estimate or to compute the exact value in a simple context. However, when they were asked to estimate values with reasonable justifications, many students failed to justify their methods of estimation. The majority of students were also unable to judge the reasonability of answers from computations by the given method of the question.

Q52/M3 Example of Student Work (Exact values were used mistakenly) 理由: **否 国为商奠公司指名 当 30元 标獲取 纪念品-**伤 3回伤74.3元的遗物、夏雾数是297.2,他 游怒而冒 主流違列而 贫公司的要求漏 300元, 的几乎不能够得纪 Example of Student Work (Explanation was not completed; Rounding-off method should not be used) 理由: F棺、玩 Y6 < \$300 Example of Student Work (Good performance) Explanation: The estimated value of the gipvice of the gift is \$75 while the exact value of the price of the gift is \$74.3 which is less than the estimated value the estimated value of the price of After both value are multiplied by 4, the second value must be less than the estimated value. Therefore, the actual amount he buys must be less than \$300. He cannot get a souvenir.

Q52/M1

Example of Student Work (Without expressing the length of copper pipe with a suitable approximation)

 $26 \div 4.22$

- 4 、最为切 4條長 4.22m 的 题 管

Example of Student Work (Could not estimate the length of copper pipe by Rounding-up method)

们们有是 Example of Student Work (Good performance) わわ

• Approximation and Errors: The majority of students could round off a number to 3 decimal places or 3 significant figures. However, their performance was fair when they were asked to find the original number from the rounded number.

Q2/M3

Exemplar Item (Find the original number from the rounded number)

After rounding off a number to 3 significant figures, the value obtained is 0.100. The number may be

- A. 0.099 53.
- B. 0.099 9.
- C. 0.099 95.
- D. 0.100 5.
- Rational and Irrational Numbers: Most of the students could demonstrate recognition of the concept of irrational numbers. Many students could represent real numbers on the number line.

Comparing Quantities

Using Percentages: Students did well in solving simple problems on growths and regarding profit percentage, but some students confused the following terms: '% off'(or in Chinese '折'), 'discount', 'discount percentage' and 'selling price'. As in previous years, students fared better on compound – interest problems than on simple – interest problems.

Q44/M2

Exemplar Item (Find discount)

The marked price of a TV set is \$12 000. If it is sold at a discount of 15%, find the discount.

Example of Student Work (Confused '% off'(or in Chinese '折') with 'discount')

買=12000(1-0,15) =10200TC +51 +12 = 10200 ×100 % 和是儿五书

Q22/M3

Exemplar Item (Problems about simple interest)

Jackson deposited \$4 000 in a bank. The simple interest rate was 2% p.a. Find the amount received after 4 years.

Example of Student Work (Calculated the interest only)

本利和是 _____ 元。

Example of Student Work (Mistakenly the sum of interest and four times the principal was taken)

The amount is \$ _____320

• Rate and Ratio: Performance was quite good. Students demonstrated good recognition of the basic concept of rate and ratio. They could represent a ratio in the form *a* : *b*. However, their performance was fair when they had to use ratio to solve simple real-life problems.

Q49/M2

Exemplar Item (Use ratio to find the weight of sand)

A mixture weighs 20 g. It is a mixture of sand and rice. The weight of sand in the mixture is 8 g.

- (a) Find the weight of sand : the weight of rice.
- (b) Michael adds an extra amount of sand to the mixture so that the ratio of the weight of sand to rice is changed to 5 : 6. How many grams of sand should he add?

Example of Student Work (Mistakenly calculated the total weight of sand)

D)設化加入文克的沙 1.(Oto) $\lambda \log \lambda$ Example of Student Work (Correct solution) to JXR _____ 是12克 _____ +4.8_____ _____ ££_____

Observing Patterns and Expressing Generality

• Formulating Problems with Algebraic Language: The majority of students could translate word phrases/contexts into algebraic languages. They could substitute values into some common and simple formulas and find the value of a specified variable. They could also write down the next few terms in sequences from several consecutive terms that were given. However, many students could not formulate simple equations from simple contexts.

Q25/M2

Exemplar Item (Formulate simple equations from simple contexts)

Karen is x years old now and her age is three times John's age. After 4 years, John will be y years old. Write down an equation to represent the relationship between x and y.

Example of Student Work (Could not distinguish between 'after 4 years' and '4 times')

Example of Student Work (Could not express 'Karen's age is three times John's age')

3x+4=y

 Manipulations of Simple Polynomials: Students could do some basic manipulations with polynomials but they still performed poorly in finding the degree of polynomials. There was room for improvement in finding the number of terms of polynomials. Furthermore, the majority of students could not distinguish polynomials from algebraic expressions.

Q26/M2

Exemplar Item (Find the number of terms of the polynomial)

Find the number of terms of the polynomial $-6+7x-5x^2+x^3$.

Example of Student Work (Mixed with number of terms and variable of the polynomial)

多項式的項數是 // 。

Example of Student Work (Probably without considering the constant term)

多項式的項數是 3___。

• Laws of Integral Indices: Students in general could find the value of a^n , where a and n are integers. Students' performance was quite good when they had to simplify algebraic expressions using the laws of integral indices.



 Factorization of Simple Polynomials: Students' performance was fair. Students could not demonstrate recognition of factorization as a reverse process of expansion. There was room for improvement in factorizing simple polynomials by taking out common factors, using the difference of two squares, the perfect square expressions and applying the cross method.

Q27/M1

Exemplar Item (Factorize the expression by using the cross method)

Factorize
$$x^2 + 3x - 4$$
.

Example of Student Work

$$x^{2}+3x-4 = (7 + 3) - 4$$

Example of Student Work

$$x^{2} + 3x - 4 = (x - 2)^{2}$$

Example of Student Work

$$x^{2}+3x-4 = (x+2)(x-2)$$

Algebraic Relations and Functions

- Linear Equations in One Unknown: Students did well in solving equations if all coefficients and constants were integers. Nevertheless, more than half of students could not solve the equations if some of the coefficients were fractions.
- Linear Equations in Two Unknowns: Many students could use algebraic or graphical methods to solve linear simultaneous equations. They could also formulate simultaneous equations from simple contexts. However, most of them were unable to draw the graph of y 3 = 0. Many students drew the graph by using the three points (-2, 1), (0, 3) and (2, 5).

Q48/M4	
Example of Student Work	y-3=0
(Could not find the correct values of <i>y</i>)	x -2 0 2 y 1 3 5 4
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Example of Student Work (Without exter	nding in two ends, a line segment was drawn
instead)	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Example of Student Work	y-3=0
(Good performance)	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

• Identities: More than half of the students were able to distinguish equations from identities. Moreover, almost half of students could use the difference of two squares and perfect square expressions to expand simple algebraic expressions respectively.

Q29/M1

Example of Student Work

$$(4+x)(4-x) = 16+\chi^2$$

Example of Student Work

$$(4+x)(4-x) = -\sqrt{2} - \sqrt{2}$$

Q29/M3

Example of Student Work (Has mistakenly taken $(a+b)^2 = a^2 + b^2$ as an identity)

$$(3x+5)^2 = -\frac{9x^2+26}{2}$$

• Formulas: Students' performance was quite good. They could find the value of a specified variable in the formula. They could also simplify algebraic fractions and perform a change of subject in simple formulas.

Q50/M3

Exemplar Item (Change of subject) $A \text{ cm}^2$ is the total surface area of a cone. A can be calculated by the following formula $A = \pi r(k + r)$, where r cm and k cm represent the base radius and the slant height of the cone respectively. (a) Make k the subject of the formula. (b) If $A = 90\pi$ and r = 5, find the value of k. k cmr cm



 Linear Inequalities in One Unknown: Students could use inequality signs to compare numbers, represent inequalities on the number line and solve simple linear inequalities in one unknown. On the other hand, almost half of the students could not formulate linear inequalities in one unknown from simple contexts and demonstrate recognition of the properties of inequalities.

Q31/M4

Exemplar Item (Solve simple linear inequalities in one unknown)

Solve the inequality 5x - 2 < 18.

4

Example of Student Work (Could not express the answer in inequality)

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S.3 Measures, Shape and Space Dimension

S.3 students performed steadily in this Dimension. They could find measures in 2-D and 3-D figures, angles related with lines and rectilinear figures, problems related to Pythagoras' Theorem and Quadrilaterals. However, more improvement could be shown in items related to definitions and 3-D figures. Comments on students' performances are provided below with examples cited where appropriate (question number x /sub-paper y quoted as Qx/My). More items may also be found in the section *General Comments*.

Measures in 2-D and 3-D Figures

• Estimation in Measurement: Students' performance was steady. Most students could choose the method that gave a more accurate reading. Many students could find the range of measures from a measurement of a given degree of accuracy, although some of them had difficulty in estimating measures with justification.

Q52/M2		
Example of Student Work (Estimate the capacity of the tank – only estimated the		
capacity of the tank by the volume of one bottle of fruit juice)		
The capacity of the tank is around \$ There are similar LAnsquare beside the surface of the left side of the tank. So that I guess the capacity of the tank is around \$		
Example of Student Work (Estimate the capacity of the tank – good performance)		
1987年5月51回转3210ml正历淹没1标3		
市 1156備居毒-250m(×5=1250m(
· 密量 大 47法 12.50 ml ·		

- Simple Idea of Areas and Volumes: Students did quite well, especially in using the formulas for circumferences, areas of circles and cylinders.
- More about Areas and Volumes: Students' performance was steady. Quite a number of students could use formulas to calculate arc lengths, areas of sectors and surface areas of right pyramids. As seen last year, they again had difficulties in using relationships between the sides and surface area of similar figures to solve problems. They were also not able to distinguish among formulas for lengths, areas, volumes by considering dimensions.

Q49/M4

Example of Student Work (Find the total surface area of the pyramid – could not calculate the area of the lateral planes correctly)

BE= IBC A BCV的面積:
= 1 10 BEXVEX 1
=5 = 12×5×±
² 30
DC BA的面積:
10×10
₹ (00
台思表面看:
(30×4) + (00
$= 120 \pm 100 = 120 (m^2)$

Learning Geometry through an Intuitive Approach

 Introduction to Geometry: In general students could identify 3-D solids from given nets. They did well in problems relating to angles. Almost half of the students could sketch a diagram of a triangular pyramid, but they performed poorly in recognition of regular polygons and concave polygons.

Q33/M1

Example of Student Work (Sketch of a triangular pyramid –using solid lines and dotted lines inappropriately)



Example of Student Work (Sketch of a triangular pyramid –using solid lines and dotted lines inappropriately)



- Transformation and Symmetry: Students' performance was good. They demonstrated recognition of basic concepts, but their performance was only satisfactory when they identified the image of a figure after a single transformation.
- Congruence and Similarity: Students' performance remained steady. They could apply the properties of congruent and similar triangles to find the sizes of angles and the lengths of sides in general. Almost half of the students could identify whether 2 triangles are congruent/similar with simple reasons.



- Angles related with Lines and Rectilinear Figures: Students did quite well and they
 remained strong in solving geometric problems. They performed more than
 satisfactorily in this learning unit except when using the relations between sides and
 angles associated with isosceles/equilateral triangles and the formulas for the sums
 of the interior angles of convex polygons to solve simple geometric problems.
- More about 3-D figures: Quite a number of students could name planes of reflectional symmetries or axes of rotational symmetries of cubes according to context of the item. Their performance was fair on items related to the nets of right prisms with equilateral triangles as bases and matching 3-D objects with various views. However, they performed poorly when asked to name the angle between a line and a plane.

Q37/M1



Learning Geometry through a Deductive Approach

- Simple Introduction to Deductive Geometry: Almost half of the students could choose the correct geometric proof. Besides, more than half of the students were able to identify altitudes of a triangle.
- Pythagoras' Theorem: Students did well. They could use Pythagoras' Theorem to • solve simple problems and determine whether the given triangles were right-angled triangles or not by using the converse of Pythagoras' Theorem.

Q46/M2
Example of Student Work (Good performance)
12 DAB & a right angle LaTren)
$\therefore AO^2 + AB^2 = DB^2 (Pyth. Thm.)$
$0.7^{2} + 2.4^{2} = 0.8^{2}$
$CB^{2} = 6.25$
fB = 2.5
i The ditetance of is 2.5 km "

Quadrilaterals: Students performed well. They could use the properties of kites and parallelograms in numerical calculations.

Learning Geometry through an Analytic Approach

Introduction to Coordinates: Many students could not match a point under rotation with the origin through 90° with its image in the rectangular coordinate plane. Besides, some students mistakenly treated the figure as a trapezium when they were asked to calculate its area.

Q49/M3
Example of Student Work (mistakenly treated the diagram ABCD as a trapezium)
$AD = \sqrt{(-2-0)^2 + [0-(-3)^2]}$
= 53
$BC = \sqrt{(1-4)^2 + (4-0)^2}$
= 5
$CD = \int (4-0)^2 + [0-(-3)]^2$
= 5 $\times ABY = (\overline{M} + 5) \times 5 = -71 / -2$
、11400面Ng 21.5 CM
Example of Student Work (Correct solution)
$ABCD \overline{MB} = (3X4)^{2} + (3X4)^{2} + (2X3)^{2} + (4X3)^{2}$
=6+6+3+6
$= 21 \text{ wrd}^2 \text{ //}$

• Coordinate Geometry of Straight Lines: Generally students could use the distance formula. They also demonstrated recognition of the conditions for parallel lines and perpendicular lines. However, only half of the students could use the mid-point formula and find the slopes of straight lines.

Exemplar Item (Find the coordinates of the mid-point of line segment)

A(3, -1) and B(-3, 5) are two points in the rectangular coordinate plane. Find the coordinates of the mid-point of line segment AB.

Example of Student Work (Mistakenly took the formula as $(x_1 + x_2, y_1 + y_2)$)

線段 AB 的中點的坐標是 (___O___, __4___)。

Example of Student Work (Mistakenly took the formula as $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$)

線段 AB 的中點的坐標是 (______,____)。

Trigonometry

 Trigonometric Ratios and Using Trigonometry: Students showed a certain degree of understanding in the use of sine, cosine and tangent ratios. They could solve simple 2-D problems involving one right-angled triangle, but their performance was only satisfactory in recognition of the ideas of bearing, gradient, the angle of elevation and the angle of depression.

Q49/M1
Example of Student Work (Solving the sides of a right-angled triangle - poor
presentation)
$\frac{\tan 70}{2.3} = \frac{B^{\circ}C}{2.3}$ = 6.3 m
Example of Student Work (Good performance)
BCLAB
··· CABC为重新主角形
$-\tan 70^\circ = \frac{BC}{2.3}$
BC = 6.3 (m)
、牆的萬度了C为6.3 M。

S.3 Data Handling Dimension

The performance of S.3 students was steady in this Dimension. They did well in items related to using simple methods to collect data, choosing appropriate diagrams/graphs to present a set of data and finding a modal class from a set of grouped data. However, performance was weak when students were asked to distinguish discrete and continuous data and find the mean from a set of grouped data. Comments on students' performance are provided below with examples cited where appropriate (question number x / sub-paper y quoted as Qx/My). More examples may also be found in the section *General Comments*.

Organization and Representation of Data

- Introduction to Various Stages of Statistics: Students' performance was steady. A considerable number of students not only could demonstrate recognition of various stages of Statistics, but also used simple methods to collect and organize data. Nevertheless, students in general didn't know how to distinguish discrete and continuous data.
- Construction and Interpretation of Simple Diagrams and Graphs: Students could choose appropriate diagrams/graphs and construct scatter diagrams to represent a set of data. However, their performance was only fair when they were asked to interpret the diagrams. For example, in Q50/M2, when students described the relationship between the marks in the two subjects, the terms 'similar', 'increasing', 'directly proportional' were used. Some students mentioned students G and H only, and few students listed the differences between the math scores and science scores.

Q50/M2

(b)

Example of Student Work (Mistakenly used 'directly proportional' to describe the relationship between the marks in the two subjects)

新局前封甸田也等写以正R.B.

Example of Student Work (Mistakenly used 'similar' to describe the relationship between the marks in the two subjects)

(b) they got the marks of Science and Mathematics is similar.



Analysis and Interpretation of data

 Measures of Central Tendency: Many students could find the arithmetic mean or mode from ungrouped data. From a set of grouped data, they fared better in finding the modal class than in finding the arithmetic mean. More than half of the students could identify sources of deception in cases of misuse of averages.

Q42/M4

Exemplar Item (find the arithmetic mean from a set of grouped data)

The following table shows the weights (kg) of 50 newborn babies.

Weight (kg)	1.5 — 2.4	2.5 — 3.4	3.5 — 4.4	4.5 — 5.4
Frequency	3	20	22	5

Find the modal class of the weights of these 50 newborn babies.

Q52/M4

Example of Student Work (Mistakenly agreed with Mary's claim) 理由:

```
: wt w + 40 + to + to + 50 + 70 + 80 + 10+ 20 + 1500 +

50 + 90 + 40 + 1000 ÷ 15

= 20 6

: 我認為多小型分說這量對1分。
```

Example of Student Work (Without explaining clearly that there were only 2 donors whose donations were more than \$200) 理由: 不同意因為只有小歌捐款差所编的款而 日本印教 326 家。 Example of Student Work (Good performance) 理由: 我不同意。因有算術平均教家臣因有有其中雨位人行捐 了 \$ \$ 1500 和 \$ 1000 所以才把平均教拉高 但其他定有捐出自约中的10-90,所以我不同 危枕的说话。

Probability

• Simple Idea of Probability: More than half of the students could compute empirical probability and calculate the theoretical probability by listing.

Q43/M1

Exemplar Item (Calculate the empirical probability)

In the 2010 World Cup Finals, Paul the Octopus predicted the winners of 8 of the football matches. Lastly, the 8 winners were predicted correctly. Find the empirical probability that Paul predicted correctly.

Example of Student Work (Without simplifying the answer)

保羅準確預測賽果的經驗概率是 ______

Example of Student Work (Without simplifying the answer)

保羅準確預測賽果的經驗概率是 _____

Example of Student Work (Gave the number of football matches that Paul predicted correctly, but not the probability)

保羅準確預測賽果的經驗概率是 ______。

Example of Student Work (Confused '100' with '100%')

保羅準確預測賽果的經驗概率是 ______。

General Comments on S.3 Student Performances

The overall performance of S.3 students was satisfactory. Performance was steady in Number and Algebra Dimension, Measures, Shape and Space Dimension and Data Handling Dimension.

The areas in which students demonstrated adequate skills are listed below:

Directed Numbers and the Number Line:

- Use positive numbers, negative numbers and zero to describe situations in daily life (e.g. Q21/M1).
- Demonstrate recognition of the ordering of integers on the number line (e.g. Q21/M3).
- Add, subtract, multiply and divide directed numbers (e.g. Q1/M2).

Numerical Estimation:

• Determine whether to estimate or to compute the exact value in a simple context (e.g. Q22/M1).

Approximation and Errors:

- Round off a number to a certain number of significant figures (e.g. Q22/M4).
- Convert numbers in scientific notation to integers or decimals (e.g. Q1/M4).

Rational and Irrational Numbers:

• Demonstrate recognition of the integral part of \sqrt{a} , where *a* is a positive integer not greater than 200 (e.g. Q3/M2).

Rate and Ratio:

Demonstrate recognition of the difference between rate and ratio (e.g. Q24/M2).

Formulating Problems with Algebraic Language:

• Describe patterns by writing the next few terms in sequences from several consecutive terms of integral values (e.g. Q26/M4).

Manipulations of Simple Polynomials:

• Multiply a monomial by a monomial (e.g. Q6/M1).

Linear Equations in One Unknown:

• Demonstrate understanding of the meaning of roots of equations (e.g. Q7/M3).

Linear Equations in Two Unknowns:

• Formulate simultaneous equations from simple contexts (e.g. Q7/M1).

Estimation in Measurement:

• Reduce errors in measurements (e.g. Q10/M1).

Introduction to Geometry:

• Make 3-D solids from given nets (e.g. Q12/M3).

Transformation and Symmetry:

• Name the single transformation involved in comparing the object and its image (e.g. Q12/M4).

Congruence and Similarity:

• Demonstrate recognition of the properties of congruent and similar triangles (e.g. Q35/M3).

Angles related with Lines and Rectilinear Figures:

• Demonstrate recognition of vertically opposite angles (e.g. Q15/M2).

More about 3-D Figures:

• Match 3-D objects built up of cubes from 2-D representations from various views (e.g. Q16/M1).

Pythagoras' Theorem:

• Use the converse of Pythagoras' Theorem to solve simple problems (e.g. Q16/M4).

Quadrilaterals:

• Use the properties of parallelograms and kites in numerical calculations (e.g. Q38/M3 and Q38/M4).

Introduction to Coordinates:

• Use an ordered pair to describe the position of a point in the rectangular coordinate plane and locate a point of given rectangular coordinates (e.g. Q16/M2).

Measures of Central Tendency:

• Find the mean, median and mode from a set of ungrouped data (e.g. Q41/M4).

Other than items in which students performed well, the Assessment data also provided some entry points to strengthen teaching and learning. Items worthy of attention are discussed below:

Manipulations of Simple Polynomials

• Distinguish polynomials from algebraic expressions (e.g. Q4/M4): Students didn't realize that monomial is polynomial. Few students chose the correct answer A. More than half of the students chose option B.

Q4/M4

Which of the following is a polynomial?

A. $3x^{2}$ B. $3^{x} - 5x + 2$ C. $\sqrt{3x^{2} + 4x - 5}$ D. $\frac{2}{x} + 7$

• Demonstrate recognition of terminologies (e.g. Q4/M1): The analysis showed that both abler and less able students didn't demonstrate recognition of the concept of degree of polynomials. Only some students chose the correct answer C. Many students chose option A.

Q4/M1

Find the degree of the polynomial $7x^3y^2 + x^2y + 8x - 12$.

- A. 3
- B. 4
- C. 5 D. 6
- Add or subtract polynomials of at most 4 terms (e.g. Q28/M4): The performance of students was weak in dealing with the addition or subtraction of like terms / unlike terms. More than half of them could not find the correct answer.

Q28/M4 Simplify $(x^2 + 4x) + (x - 4x^2)$. Example of Student Work (Mistakenly to perform $x^2 + x = x^3$ and $4x + 4x^2 = 16x^3$) $(x^2 + 4x) + (x - 4x^2) = \underbrace{\chi^3 + \lfloor \zeta_{\chi} \rfloor}_{\text{Example of Student Work}}$ (Mistakenly to perform $(x^2 + 4x) + (x - 4x^2) = (x^2 + 4x)(x - 4x^2)$) $(x^2 + 4x) + (x - 4x^2) = \underbrace{\chi^2 (-4\chi^2 + 4 - 15\chi)}_{\text{Example of Student Work}}$

• If all the variables and indices were the same in the expression and options were provided in the question (e.g. Q6/M2), the majority of students were able to choose the correct answer.

Q6/M2 Simplify $5m^3 - 2m^3$. A. 3mB. $3m^3$ C. 3D. $\frac{5}{2}$

Identities

• Tell whether an equality is an equation or an identity (e.g. Q8/M1): Half of the students chose the correct answer D. Some students chose option A, showing that they took $(x-a)^2 = x^2 - a^2$ as an identity.

Q8/M1

Which of the following is an identity?

A. $x^2 - 4 = (x - 2)^2$ B. $x^2 - 4 = 0$ C. $5(x^2 - 4) = 5x^2 - 4$ D. $(2 + x)(x - 2) = x^2 - 4$

Linear Inequalities in One Unknown

• Demonstrate recognition of the properties of inequalities (e.g. Q9/M2): Only half of the students chose the correct answer B. Quite a number of students thought that the inequalities in options C and D were correct.

Q9/M4

If $x \le y$, which of the following inequalities **MUST** be correct? A. $2x \ge 2y$ B. $\frac{x}{-2} \ge \frac{y}{-2}$ C. $x-2 \ge y-2$ D. $-x-2 \le -y-2$

More about Areas and Volumes

• Use the relationships between sides and surface areas/volumes of similar figures to solve related problems (e.g. Q10/M3): Some students treated the relationship between sides of two similar figures the same as the relationship between their surface areas and so they chose option C.



Distinguish among formulas for lengths, areas and volumes by considering dimensions (e.g. Q11/M4): Only some students chose the correct answer D. Almost half of the students chose option B or C.



Introduction to Geometry

• Demonstrate recognition of common terms in geometry (e.g. Q11/M3): Quite a number of students chose option A or B, indicating that they thought Solid *I* was a regular polyhedron.



• Determine whether a polygon is regular, convex, concave, equilateral or equiangular (e.g. Q34/M3): Generally students didn't choose Figure C and D. However, some students also thought that diagrams in Figure B and E were not concave polygons.



Transformation and Symmetry

• Identify the image of a figure after a single transformation (e.g. Q13/M4): Only a small number of students chose the correct answer C. Almost half of the students chose option A.



Simple Introduction to Deductive Geometry

• Identify medians, perpendicular bisectors, altitudes and angle bisectors of a triangle (e.g. Q16/M3): More than half of the students chose the correct answer D. Some students confused perpendicular bisectors with altitudes of a triangle.



Introduction to Coordinates

• Match a point under a single transformation with its image in the rectangular coordinate plane (e.g. Q15/M1): Students' performance was only fair with items related to the single transformation including rotation. Almost the same number of students chose option A and option B respectively. Some students confused rotation with reflection.

Q15/M1

In the figure, point A(-4, -3) is rotated about the origin O through 90° in clockwise direction to the point A', the coordinates of A' are y 5 (-3, 4). A. 4 Β. (-4, 3). 3 2 C. (3, -4). 1 D. (4,−3). > x-4 -3 -2 -101 2 ł 3 4 $\mathbf{x}^{A(-4, -3)} \mathbf{x}^{-2}$

Trigonometric Ratios and Using Trigonometry

Demonstrate recognition of the ideas of bearing, gradient, the angle of elevation and the angle of depression (e.g. Q18/M4): Almost half of the students chose option C. They were not aware that the requirement of the question was 'O from P' instead of 'P from O'.



Introduction to Various Stages of Statistics

• Distinguish discrete and continuous data (e.g. Q19/M2): Only some students chose the correct answer B. Almost half of the students mistakenly assumed that continuous data was a natural number and so they chose the option D.

Q19/M2

Which of the following data is continuous?

- A. The number of compact discs in a drawer
- B. The weight of a steak
- C. The number of passengers in a bus
- D. The number on a restaurant queue ticket

Construction and Interpretation of Simple Diagrams and Graphs

• Read information (including percentiles, quartiles, median) and frequencies from diagrams/graphs (e.g. Q20/M2): Only about half of the students chose the correct answer A. Some students mistakenly took upper quartile to be lower quartile.

Q20/M2			
The following stem-and-leaf diagram shows the study hours of 20 students in Applied			
Learning Courses.			
Study hours of 20 students in Applied Learning Courses			
	Stem (10 hours)	Leaf (1 hour)	
	1	2 5 9	
	2	0 2 2	
	3	0 0 0 1 1 4 9	
	4	28899	
	5	09	
According to the above diagram, which of the following is correct?			
А.	A. median = 31 , lower quartile = 22		
B.	B. median = 31 , lower quartile = 48		
C.	C. median = 30 , lower quartile = 22		
D.	D. median = 30 , lower quartile = 48		

Best performance of S.3 Students in TSA 2011

Students sitting for each sub-paper were ranked according to their scores and the performances of approximately the top 10% were singled out for further analysis. The performances of these students are described below.

Among these students, more than half of them achieved a full score or lost at most seven score points in the whole assessment. They demonstrated almost complete mastery of the concepts and skills assessed by the sub-papers they attempted.

Most of these students were able to name the single transformation involved in comparing the object and its image (e.g. Q13/M1), use the formulas for circumferences and areas of circles (e.g. Q46/M1), add, subtract, multiply and divide directed numbers (e.g. Q1/M1), formulate simultaneous equations from simple contexts (e.g. Q7/M1), demonstrate recognition of vertically opposite angles (e.g. Q15/M2), use Pythagoras' Theorem to solve problems (e.g. Q46/M2), and demonstrate recognition of the integral part of \sqrt{a} (e.g. Q3/M2). They were also able to add or subtract polynomials of at most 4 terms (e.g. Q6/M2), demonstrate recognition of the properties of congruent and similar triangles (e.g. Q35/M3), make 3-D solids from given nets (e.g. Q12/M3), translate contexts into algebraic languages (e.g. Q38/M3), use the properties of parallelograms and kites in numerical calculations (e.g. Q38/M3), and use the converse of Pythagoras' Theorem to solve problems (e.g. Q6/M4), solve simple selling problems (e.g. Q44/M4), convert numbers in scientific notation to integers or decimals (e.g. Q1/M4), and solve a system of linear simultaneous equations by algebraic methods (e.g. Q46/M4).

The examples of work by these students are illustrated below:

Students with the best performance could set up and solve the problem correctly with a complete solution.

Q46/M1
Example of Student Work (Solve the radius and area of a circle)
a) Let the radius of the circle be r:
$2r\pi = 28\pi$
r= 14
The Value of ris 14
b) the area of the circle:
$= \gamma^2 \pi$
$= 196 \text{ TV} (\text{cm}^2)$
The area of the circle is 196 Tu(cm)

Q49/M1
Example of Student Work (Solve the height of a wall)
BCLAB
、CABC为重角之角形
$-\tan 70^\circ = \frac{BC}{2.3}$
BC = 6.3 (m)
、牆的為度BC为6.3m。

Students with the best performance could make good use of the given conditions and solve the problem systematically.

Q50/M3
Example of Student Work (Using the result of (a) to find k in part (b))
$(a) A = \pi r'(k+r)$
$k = \frac{A}{\pi r} - r$
(b) Listitute A=90x and r=5 into k=2r-r
$k = \frac{1}{2} v - r$
$k = \frac{1}{2\sqrt{5}} - \frac{1}{5}$
$k = -\frac{1}{2} - \frac{1}{2}$
k = 13

Students with the best performance could show steps clearly and used correct reasoning to set up the conclusion.

Q52/M1

Example of Student Work (Find the number of segments that can be cut from copper pipe)

把14,22m着作4m	最刻切出铜管数目=2074
	二5係)
把4.22m着作:5m	最多可切出铜管数月= 2075
	二4(条)
' 5m	74.22m79m
、沈晨多	5可切出铜管数图为4采。

Q52/M1

Example of Student Work (Estimate the capacity of the tank)

Men the juice in the bottle is completely poured into the task, the rise in water level is AB3, covering a square of the task. Ne just need to pour the juice into the task by using the same bottle (some apacity). As each time after pouring the juice, the rater level will rise on one guare So, it is estimated that we need to pour 5 traves [abolled] as there is 5 squares on the task S., the capacity of the task. is sto x t = 1250 mL.

Q52/M3

Example of Student Work (Judge whether Jack's estimation is reasonable)

当計算化應 3 獲得紀名品时,應把数畫量計 力,不應加大,如果 偉 傑要計 到他最小購物 都多於 招加, 就能證明 化-定態得到 紀名品, 但如果 把 個 錢 加大, 教 不能 證 复化一定能得 到 紀名 品。另外, 化 計出每件 礼 品 早 75 太 刚 刚 正約 購 約, 南 丰 50, 但 化 夏的礼 品 都不足 丰 55, 所以 化一定 7% 得到, 紀 名 品。

Q52/M4

Example of Student Work (Judge whether Mary's claim is reasonable)

No, first of all, from the table we can see that more of

the donors donote below & 100, mly two of them donote.

more than it 1000. S that although the mean of the

- donution 13 \$206, we can clearly notice that is just
- because the two of them who owned I woo and dibo whom

mule the mean tigner. Therefore, we surely cannot use

the mean to conclude that most of the donors anded

more that \$200.

Some common weak areas of high-achieving students are listed as follows:

- Some students could not demonstrate recognition of degree of a polynomial.
- Some students could not judge, without actual calculations, the reasonability of answers from computations.
- Quite a number of students could not plot the graph of the equation y-3=0.
- Quite a number of students could not distinguish polynomials from algebraic expressions.

Comparison of Student Performances in Mathematics at Secondary 3 TSA 2009, 2010 and 2011

This was the sixth year that Secondary 3 students participated in the Territory-wide System Assessment. The percentage of students achieving Basic Competency this year was 80.1% which was about the same as last year.

The percentages of students achieving Basic Competency from 2009 to 2011 are listed below:

Year	% of Students Achieving Mathematics Basic Competency
2009	80.0
2010	80.1
2011	80.1

Table 8.7Percentages of S.3 Students Achieving Mathematics Basic
Competency from 2009 to 2011

The performances of S.3 students over the past three years in each Dimension of Mathematics are summarized below:

Number and Algebra Dimension

- Directed Numbers and the Number Line: The performance of students has been good over the past three years.
- Numerical Estimation: Students were able to determine whether to estimate or to compute the exact value in a simple context, but there was room for improvement of students' performance when students were asked to estimate values with reasonable justifications.
- Approximation and Errors: Performance was steady on conversion of significant figures. Students did well when they converted numbers in scientific notation to integers.
- Rational and Irrational Numbers: Performance remained steady. In representing real numbers on the number line, the performance of students was better compared with past years.
- Using Percentages: Students could not master the concept of percentage (e.g. they confused discount with discount rate) and units were often omitted in the

answer. Performance remained fair in solving problems about simple interest.

- Rate and Ratio: Recognition of the difference between rate and ratio improved. On the other hand, the presentation of students' answers remained unclear and incomplete in solving simple real-life problems.
- Formulating Problems with Algebraic Language: The performance of students remained good in items involved numerical calculation. Solving the problems involving the n^{th} term of number sequence were still the weak spots.
- Manipulations of Simple Polynomials: Students were still weak in distinguishing polynomials from algebraic expressions. Performance was only fair in recognition of terminologies. They performed steadily in addition, subtraction or multiplication of polynomials.
- Laws of Integral Indices: Performance was steady in using the laws of integral indices to simplify simple algebraic expressions.
- Factorization of Simple Polynomials: Students were weak in recognition of factorization as a reverse process of expansion. Performance remained satisfactory in factorization of simple polynomials by taking out common factors, using the difference of two squares, the perfect square expressions or the cross method.
- Linear Equations in One Unknown: The performance of students was good in understanding the meaning of roots of equations, but they regressed on items related to formulation of linear equations in one unknown from simple contexts.
- Linear Equations in Two Unknowns: They did quite well in plotting graphs of linear equations ax + by + c = 0, but they were weak in recognition of the graphs of my + n = 0. Nevertheless, they were more able to formulate simultaneous equations from simple contexts. They also showed improvement in recognition of the graphs of equations of the form ax + by + c = 0 are straight lines. Besides, students didn't favor using the method of substitution to solve simultaneous equations.
- Identities: Performance of students was fair in recognition of identity. Performance was steady when they were asked to expand simple algebraic expressions by using the difference of two squares and the perfect square expressions.

- Formulas: Students were more capable of manipulating algebraic fractions and performing change of subject in simple formulas this year.
- Linear Inequalities in One Unknown: Performance of students was fair in recognition of the properties of inequalities. They fared better in representing inequalities on the number line than representing the solution on the number line by formulating linear inequalities.

Measures, Shape and Space Dimension

- Estimation in Measurement: Generally students could choose appropriate methods to reduce errors in measurements. However, they did not perform as well in choosing an appropriate unit and the degree of accuracy. The performance was similar to last year in items relating to choosing an appropriate measuring tool and technique and estimating measures with justification.
- Simple Idea of Areas and Volumes: Students performed steadily in calculation involving radius, circumference, area of circle and surface areas of simple solids. They did quite well in applying formulas for volumes.
- More about Areas and Volumes: Performance was steady. In particular, their performance remained unsatisfactory in items dealing with relationships of sides and surface areas in similar figures and distinguishing among formulas for lengths, areas, volumes by considering dimensions.
- Introduction to Geometry: Performance remained good in identifying the relationship between 3-D solids and their nets. However, they were still weak in recognition of polygons and common terms in geometry.
- Transformation and Symmetry: Performances of students were quite good in general, although they performed fairly on the problems involving rotation.
- Congruence and Similarity: Students could use the properties of congruent and similar triangles to solve triangles. However, they confused the conditions for congruent triangles with similar triangles.
- Angles related to Lines and Rectilinear Figures: Students showed good recognition
 of angles with respect to their positions relative to lines. They could also use the
 angle properties associated with intersecting lines/parallel lines and the properties
 of triangles to solve simple geometric problems.

- More about 3-D Figures: Students' performance was quite good in matching 3-D objects from 2-D representations this year. Performance was still weak when dealing with the angle between a line and a plane and the angle between 2 planes.
- Simple Introduction to Deductive Geometry: Performance was steady. Students were unfamiliar with some theorems and so they usually could not give correct reasons to perform simple proofs.
- Pythagoras' Theorem: Students were capable of applying Pythagoras' Theorem to solve simple problems.
- Quadrilaterals: Performance remained good. Students could use the properties of quadrilaterals to solve geometric problems.
- Introduction to Coordinates: They demonstrated good recognition of the coordinates system. However, they were still weak in items relating to calculating areas of simple figures and transformation.
- Coordinate Geometry of Straight Lines: Performance in applying the formula of slope and mid-point formula remained fair.
- Trigonometric Ratios and Using Trigonometry: Performance remained steady in general. Students could solve simple 2-D problems involving one right-angled triangle.

Data Handling Dimension

- Introduction to Various Stages of Statistics: Performance was steady. Students were still weak in distinguishing between discrete and continuous data.
- Construction and Interpretation of Simple Diagrams and Graphs: Performance was steady in constructing simple statistical charts. However, they didn't do well in interpreting simple statistical charts, reading information from diagrams/graphs and identifying sources of deception in misleading graphs.
- Measures of Central Tendency: The item related to finding mean from a set of grouped data was still the weak area for students, but students performed fairly in other items.
- Simple Idea of Probability: Students' performance regressed in calculating the empirical probability this year. Besides, their performance varied in calculating the theoretical probability by listing the past 3 years.

Comparison of Student Performances in Mathematics at Primary 3, Primary 6 and Secondary 3 TSA 2011

The percentages of P.3, P.6 and S.3 students achieving Basic Competency from 2004 to 2011 are as follows:

Year	% of Students Achieving Mathematics BC										
Level	2004	2005	2006	2007	2008	2009	2010	2011			
P.3	84.9	86.8	86.9	86.9	86.9	#	87.0	87.0			
P.6		83.0	83.8	83.8	84.1	#	84.2	84.1			
S.3			78.4	79.9	79.8	80.0	80.1	80.1			

 Table 8.8
 Percentages of Students Achieving Mathematics Basic Competency

Due to Human Swine Influenza causing the suspension of primary schools, the TSA was cancelled and no data has been provided.

A comparison of strengths and weaknesses of P.3, P.6, and S.3 students in TSA enables teachers to devise teaching strategies and tailor curriculum planning at different key stages to adapt to students' needs. The dimensions of Mathematics Curriculum at each key stage belong to different dimensions as shown below:

Table 8.9 Dimensions of Mathematics Curriculum for Primary 3, Primary 6 and
Secondary 3

	Primary 3	Primary 6	Secondary 3		
Dimension	Numbor	Number			
	INUITIDEI	Algebra			
	Measures	Measures	Measures, Shape and Space		
	Shape and Space	Shape and Space			
	Data Handling	Data Handling	Data Handling		

The following table compares students' performances at P.3, P.6 and S.3 in Mathematics TSA 2011:

Level	P.3	P.6	S.3
Dimension			
Number	• Most P.3 students were capable of recognising the place values in whole numbers and the value of the digit.	• P.6 students were capable of recognising the place value of digits in whole numbers and decimals.	• Students understood directed numbers and their operations.
	 Majority of the P.3 students could perform arithmetic calculations with numbers up to 3 digits. P.3 students were capable of comparing fractions and recognising the relationship between fractions and the whole. Some students could not fully understand the basic concept of a fraction as parts of one whole and the relationship between fractions and the whole. 	 The majority of the P.6 students could perform arithmetic operations on whole numbers, fractions and decimals. Some P.6 students forgot the computational rule of "performing multiplication/division before addition/subtraction" when carrying out mixed operations. P.6 students performed better than P.3 students when comparing fractions and recognising the 	 Students could operate the number line. The majority of students could demonstrate recognition of the concept of irrational numbers. Students could use rate and ratio to solve the problems.
		relationship between fractions and the whole.	
	N. A.	 Many students were capable of choosing the appropriate mathematical expression to estimate the value for specific situations. 	• The majority of students were capable of choosing a reasonable expression to estimate answers from computations. However, their performance was fair when they explained the strategies.
	• Majority of the P.3 students could solve simple application problems by presenting clear working steps and explanations. Students were able to perform division of money not involving conversion of money.	• The majority of the P.6 students could solve application problems by using logical steps and clear explanations. Some students had difficulty in solving application problems involving fractions or percentages.	• Students did well in using percentages to solve simple application problems. Some students mixed up the concepts (e.g. discount and discount %) so that they could not present their steps correctly.

Table 8.10 C	omparison	of Student	Performa	nces in	Mathem	atics at	t Primary	3	, Primar	y 6 and	l Secondar	y 3	TSA	201	1
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Level Dimension	P.3	P.6	S.3
Algebra		• P.6 students were capable of using symbols to represent numbers.	• Performance of students was only fair in recognition of terminologies.
		 They were capable of solving equations involving at most two steps in the solutions. Many P.6 students managed application 	• Students were capable of solving simple equations. They could also substitute values into formulas to find the unknown value.
	NI A	problems by solving simple equations.	• Many students could solve a system of simple linear simultaneous equations by using algebraic methods or by using the graphical method.
	N. A.		• Students' performance was fair when they had to simplify algebraic expressions by using the laws of integral indices.
			• Students didn't demonstrate recognition of and apply the properties of inequalities.
			 Students' performance remained fair in factorization and expansion of simple polynomials.

Level	P.3	P.6	S.3
Measures	 The majority of P.3 students could identify and use Hong Kong Money. P.3 students performed well in exchanging money but had difficulty when the amount was 	 P.6 students could apply the basic concepts of time, length, distance, weight and capacity to simple situations. The majority of students could measure or the basic could measure or the basic	 Most students were able to choose the method that gave a more accurate reading. Students could calculate arc lengths and
	 large or they were required to do simple calculations before exchanging money. P.3 students in general were able to tell the time on a clock face and a digital clock. P.3 students had difficulty in indentifying the start date/end date of an event with a given duration of an activity. 	 calculate the perimeters and areas of simple 2-D shapes. P.6 students were capable of applying the formula of circumference. The majority of students could find the volume of cubes and cuboids but were weak in understanding the relationship between capacity. 	 Most students could use the formulas for surface areas and volumes of simple solids.
	B2 students could measure and compare the	 P.6 students could solve simple problems involving speed. 	
	 P.3 students could measure and compare the length of an object using millimetres or centimetres. P.3 students could measure and compare the weight of objects using grams and kilograms. P.3 students could measure the capacity of containers using litres or milliliters but were weak in comparing the capacity of containers. 	 P.6 students could measure length, weight and capacity with appropriate tools. P.6 students were capable of choosing the appropriate unit of measurement for recording length, weight and capacity. 	 Most students could estimate measures. They could also give simple explanations of their estimations. Students were weak in more abstract concepts (such as using relationship of
		• P.6 students performed better than P.3 students when comparing the weight of objects using improvised units.	similar figures to find measures, and the meaning of dimensions).

Level	P.3	P.6	S.3
Shape and Space	 P.3 students were capable of identifying 2-D and 3-D shapes when these shapes are drawn in a commonly seen orientation. P.3 students could recognise the simple characteristics of triangles, including right-angled triangles, isosceles triangles and equilateral triangles. P.3 students did well in comparing the sizes of angles and recognizing right angles. P.3 students could identify straight lines and curves. P.3 students performed well in recognizing the four directions. 	 P.6 students did better than P.3 students in identifying parallel lines and perpendicular lines. P.6 students were capable of identifying and grouping 2-D and 3-D shapes in different orientations. P.3 students were only required to recognize the four directions whereas P.6 students had to know the eight compass points. P.6 students performed better than P.3 students in applying their knowledge of directions to solve problems. 	 Students could identify types of angles with respect to their sizes. Students could identify the relation between simple 3-D solids and their corresponding 2-D figures. They could also sketch cross-sections of simple solids. Many students could not identify the angle between a line and a plane and the angle between 2 planes. Students could deal with simple symmetry and transformation. Students sometimes confused the conditions for congruent triangles with conditions for similar triangles. Students had good knowledge of the rectangular coordinate system. However, their performance was only fair when they had to manipulate problems involving transformation. Students performed well in numerical calculations of simple geometric problems. Students could use the angle properties associated with intersecting lines/parallel lines and the properties of triangles to solve simple geometric problems.

Level	D.4		
Dimension	P.3	P.6	S.3
Data Handling	 P.3 students performed well at reading simple pictograms with one-to-one representation. They could answer straightforward questions by retrieving data and explain their reasoning using the data given from the pictogram P.3 students could construct pictograms using one-to-one representation and give a suitable title. 	 P.6 students performed well at reading pictograms and bar charts. They could extract data to interpret the given information. P.6 students could construct statistical graphs from given data. A few P.6 students could not draw bar charts to correct scale. A small number of P.6 students mistakenly added a 'frequency axis' to a pictogram. P.6 students were capable of finding the average of a group of data and solving simple problems of averages. 	 Students understood the basic procedures of statistical work. They could collect data using simple methods. Many students could not distinguish discrete and continuous data. Students could construct simple statistical charts, but they were weak in interpreting the information. Students could choose appropriate diagrams/graphs to present a set of data. However, their performance was only fair when they read information from diagrams/graphs. Many students could explain the reason of deception in their own words in cases of the misuse of averages. Students could calculate averages from ungrouped data. However, they did not do as well when using grouped data. Students' performance was fair in calculating the empirical probability and calculating the theoretical probability by listing.