Results of Secondary 3 Mathematics in TSA 2012

The territory-wide percentage of S.3 students achieving Mathematics Basic Competency in TSA 2012 was 79.8%. The proportion achieving basic competency in 2012 was almost the same as that of last year.

Secondary 3 Assessment Design

The design of assessment tasks for S.3 was based on the documents *Mathematics Curriculum: Basic Competency for Key Stage 3 (Tryout Version)* and *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5), 1999*. The tasks covered the three dimensions of the mathematics curriculum, namely **Number and Algebra**, **Measures, Shape and Space**, and **Data Handling**. They focused on the Foundation Part of the S1 – 3 syllabuses in testing the relevant concepts, knowledge, skills and applications.

The Assessment consisted of various item types including multiple-choice questions, fill in the blanks, answers-only questions and questions involving working steps. The item types varied according to the contexts of the questions. Some test items consisted of sub-items. Besides finding the correct answers, students were also tested in their ability to present solutions to problems. This included writing out the necessary statements, mathematical expressions and explanations.

The Assessment consisted of 164 test items (221 score points), covering all of the 129 Basic Competency Descriptors. These items were organized into four sub-papers, each 65 minutes in duration and covering all three Dimensions. Some items appeared in more than one sub-paper to act as inter-paper links. Each student was required to attempt one sub-paper only.

The composition of the sub-papers was as follows:

Table 8.3 Composition of the Sub-papers

Sub-paper	Number of Items (Score Points)			
	Number and Algebra Dimension	Measures, Shape and Space Dimension	Data Handling Dimension	Total
M1	23 (32)	23 (31)	5 (7)	51 (70)
M2	24 (32)	22 (30)	5 (8)	51 (70)
M3	24 (33)	21 (28)	6 (9)	51 (70)
M4	24 (32)	21 (28)	6 (9)	51 (69)
Total *	75 (100)	71 (96)	18 (25)	164 (221)

* Items that appeared in more than one sub-paper are counted only once.

The item types of the sub-papers were as follows:

Table 8.4 Item Types of the Sub-papers

Section	Percentage of Score Points	Item Types
A	~ 30%	• Multiple-choice questions: choose the best answer from among four options
B	~ 30%	• Calculate numerical values • Give brief answers
C	~ 40%	• Solve application problems showing working steps • Draw diagrams or graphs • Open-ended questions requiring reasons or explanations

Performance of S.3 Students with Minimally Acceptable Levels of Basic Competence in TSA 2012

S.3 Number and Algebra Dimension

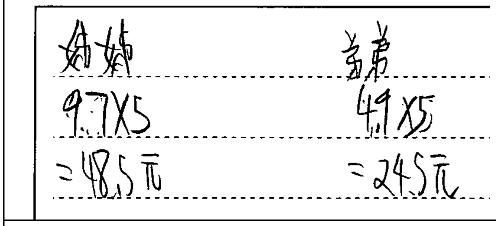
The performance of S.3 students was quite good in this Dimension. The majority of students demonstrated recognition of the basic concepts of directed numbers, approximation and errors, rational and irrational numbers, rate and ratio, laws of integral indices and equations. Performance was only satisfactory in items related to factorization of simple polynomials. Comments on students' performances are provided below with examples cited where appropriate (question number *x* / sub-paper *y* quoted as Q*x*/M*y*). More examples may also be found in the section ***General Comments*.**

Number and Number Systems

- Directed Numbers and the Number Line: Most students could demonstrate recognition of the ordering of integers on the number line. They could also handle the simple operation of directed numbers.
- Numerical Estimation: Students were able to determine the values mentioned in a simple context that were exact or estimated. More than half of the students could estimate values with reasonable justifications. Moreover, many students were able to judge the reasonability of answers from computations by the given information of the question.

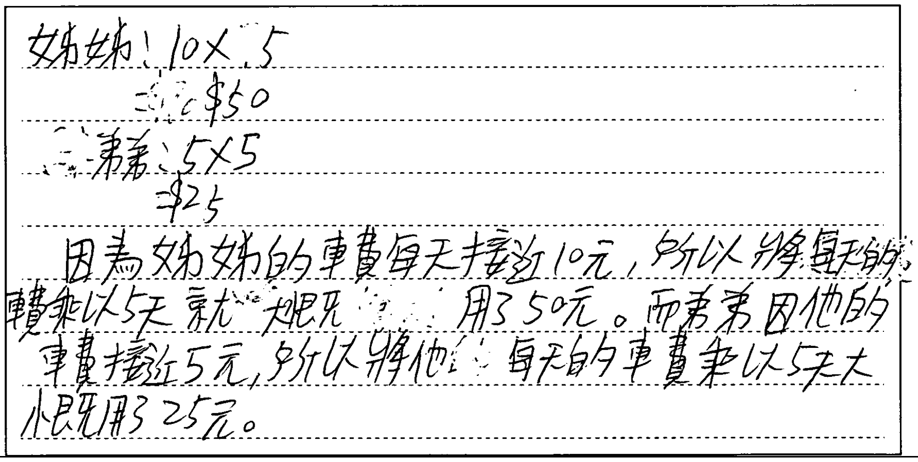
Q51/M2

Example of Student Work (Estimate the bus fares – Without using approximation for estimation)



姊姊　　弟弟
9.7X5　　4.9X5
=48.5元　　=24.5元

Example of Student Work (Good performance)



姊姊：10×5
=$50
弟弟：5×5
=$25
因為姊姊的車費每天接近10元，所以將每天的車費乘以5天就大概用了50元。而弟弟因他的車費接近5元，所以將他每天的車費乘以5天大概用了25元。

- Approximation and Errors: The majority of students could round off a number to 3 significant figures, represent a number in scientific notation and convert numbers in scientific notation to integers or decimals. However, their performance was fair when they were asked to round off a number to 3 decimal places.

Q21/M4

Exemplar Item (Round off a number to 3 decimal places)

Round off 0.026 68 to 3 decimal places.

Example of Student Work (Mistakenly took 3 decimal places as showing a number with 3 digits in the answer)

0.03

Example of Student Work (Confused 'round off a number to 3 decimal places' with 'round off a number to 3 significant figures')

0.0267

- Rational and Irrational Numbers: The majority of students could demonstrate recognition of the concept of irrational numbers and represent real numbers on the number line.

Comparing Quantities

- Using Percentages: Students did well in solving problems regarding marked price and discount percentage, however, their performance was fair when they were asked to find the cost price. Students fared better on compound – interest problems than on simple – interest problems. Furthermore, many students could solve simple problems on growths and depreciations.

Q43/M4
Exemplar Item (Find annual interest rate) Daniel deposits $9 600 in a bank. After half a year, he will obtain a simple interest of $144. Find the annual interest rate.
Example of Student Work (Confused *y* with *y*%) 設年利率為 y % $9600 \times \frac{1}{2} \times y = 144$ $4800y = 144$ $y = 0.03$ ∴ 年利率是 0.03 %
Example of Student Work (Without expressed the answer in percent) 設年利率為 x $9600 \times x = 144 \times 2$ $9600 \times x = 288$ $x = 288 \div 9600$ $x = 0.03$
Example of Student Work (Good performance) 設年利率為 x % $9600 \times \frac{x}{100} \times \frac{1}{2} = 144$ $\frac{x}{100} = 0.03$ $x = 3$ ∴ 年利率是 3 %

- Rate and Ratio: Performance was quite good. Students demonstrated good recognition of the basic concept of rate and ratio. They could find the other quantity from a given ratio *a* : *b* and the value of either *a* or *b*. They performed well when they had to use rate and ratio to solve simple real-life problems.

Q24/M2

Exemplar Item (Find the ratio of the ages)

Calvin and Tim are 15 and 21 years old respectively. Find the ratio of their ages after 3 years.

Example of Student Work (Mistakenly calculated the ratio of their present ages)

3 年後偉傑的年齡：3 年後大雄的年齡 = 5 : 7

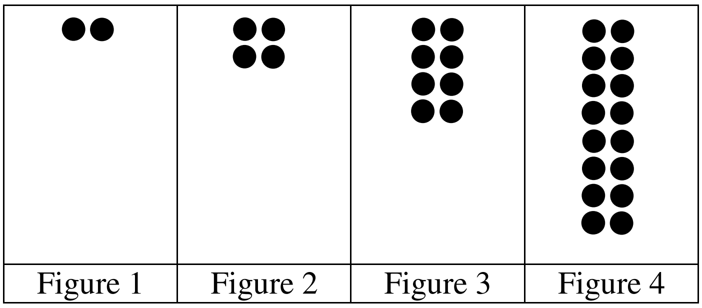
Observing Patterns and Expressing Generality

- Formulating Problems with Algebraic Language: Most students could translate word phrases/contexts into algebraic languages. They could substitute values into some common and simple formulas and find the value of a specified variable. They could also write down the next few terms in sequences from several consecutive terms that were given. Quite a number of students could formulate simple equations from simple contexts. Nevertheless, many students could not intuitively find the n^{th} term of a simple number sequence.

Q25/M1

Exemplar Item (Find the n^{th} term of a number sequence)

The following figures are formed by 2, 4, 8 and 16 circles respectively.



According to the above pattern, Figure *n* is formed by ______________ circles.

Example of Student Work (Could not distinguish between $2n$ and 2^n .)

圖 *n* 由 2n 個圓形組成。

Example of Student Work (Mistakenly took the number of circles in Figure 5 to be the number of circles in Figure *n*)

圖 *n* 由 32 個圓形組成。

- Manipulations of Simple Polynomials: Students could do some basic manipulations with polynomials. More than half of the students could distinguish polynomials from algebraic expressions. However, they still performed poorly in recognizing the variable of a polynomial.

Q26/M3

Exemplar Item (Write down the variable of a polynomial)

Write down the variable of the polynomial $7x^6 - x^2 + 5x + 8$.

Example of Student Work (Confused the variable with coefficient)

多項式的變數是 ___5___ 。

Example of Student Work (Confused the variable with constant)

多項式的變數是 ___8___ 。

- Laws of Integral Indices: The performance of students was fair in finding the value of a^n, where a and n are integers. They did quite well when they had to simplify algebraic expressions using the laws of integral indices.

Q45/M4

Example of Student Work (Has mistakenly taken $x^m \cdot x^n = x^{mn}$)

(b) $\frac{(X^3)^2}{X^{-5}}$

$= \frac{X^6}{X^{-5}}$

$= \frac{X^6}{\frac{1}{X^5}}$

$= X^6 \div \frac{1}{X^5}$

$= X^6 \times X^5$

$= X^{30}$

Example of Student Work (Has mistakenly taken $(x^m)^n = x^{m+n}$)

a) $(x^3)^2$

$= (x^{3+2})$

$= x^5$ //

- Factorization of Simple Polynomials: As in previous years, students' performance was fair in factorizing simple polynomials by taking out common factors, using the difference of two squares, the perfect square expressions and applying the cross method.

Q28/M4

Exemplar Item (Factorize the expression by using the cross method)

Factorize $x^2 - x - 12$.

Example of Student Work

$x^2 - x - 12 =$ $(x-6)(x+2)$

Example of Student Work

$x^2 - x - 12 =$ $(x+4)(x-3)$

Algebraic Relations and Functions

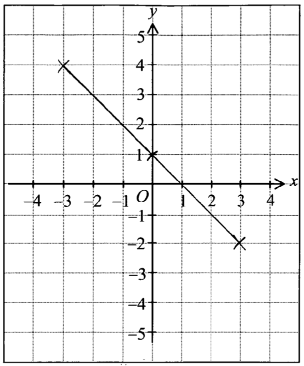
- Linear Equations in One Unknown: The majority of students could formulate linear equations in one unknown from simple contexts. Nevertheless, their performance was fair only in solving equations when the bracket is involved and the coefficient of a term is a negative number.
- Linear Equations in Two Unknowns: Many students could use algebraic or graphical methods to solve linear simultaneous equations. They could also demonstrate recognition that graphs of equations of the form *ax* + *by* + *c* = 0 are straight lines. However, most of them were unable to draw the graph of *y* = 4.

Q46/M1

Example of Student Work (Could not find the correct values of *y*)

$y = 4$

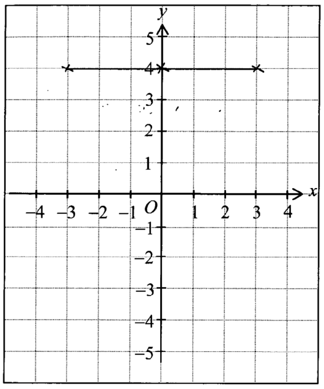
x	-3	0	3
y	4	1	-2



Example of Student Work (Without extending in two ends, a line segment was drawn instead)

$y = 4$

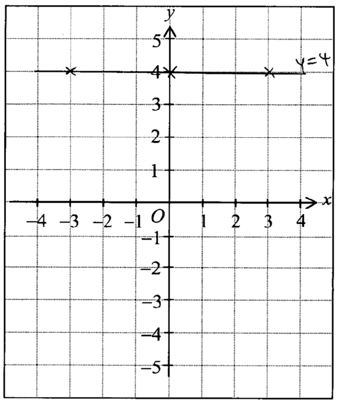
x	-3	0	3
y	4	4	4



Example of Student Work (Good performance)

$y = 4$

x	-3	0	3
y	4	4	4



- Identities: A considerable number of students were able to distinguish equations from identities. Moreover, more than half of the students could use the difference of two squares or perfect square expressions to expand simple algebraic expressions.

Q29/M1

Example of Student Work (Has mistakenly taken $(x+a)(x-a) = x^2 - 2ax + a^2$ as an identity)

$(x+6)(x-6) = x^2 - 12x + 36$

Example of Student Work (Without expressing $x^2 - 6^2$ as $x^2 - 36$)

$(x+6)(x-6) = x^2 - 6^2$

Q30/M2

Example of Student Work (Has mistakenly taken $(a+b)^2 = (a+b)(a-b)$ as an identity)

$(x+2)^2 =$ $(x+2)(x-2)$

Example of Student Work (Has mistakenly taken $(a+b)^2 = a^2 + b^2$ as an identity)

$(x+2)^2 =$ x^2+4

- Formulas: Students generally could find the value of a specified variable in the formula. Their performance was fair in simplifying algebraic fractions and performing a change of subject in simple formulas.

Q46/M2

Exemplar Item (Change of subject)

Degree Fahrenheit (°*F*) and degree Celsius (°*C*) are two kinds of units for measuring temperature. The relation between *F* degree Fahrenheit and *C* degree Celsius can be represented by the following formula:

$$F = \frac{9C}{5} + 32$$

(a) Make *C* the subject of the formula.

(b) If *F* = 104, find the value of *C*.

Example of Student Work

(a) $F = \frac{9c}{5} + 32$ [c]

$5F = 9c + 32$

$\frac{5F}{c} = 9 + 32$

$c = \frac{9}{5F} + 32$

(b) $F = 104$

$104 = \frac{9c}{5} + 32$

$104 - 32 = \frac{9c}{5}$

$72 = \frac{9c}{5}$

$360 = 9c$

$C = 40$

- Linear Inequalities in One Unknown: Many students could use inequality signs to compare numbers, formulate linear inequalities in one unknown from simple contexts and demonstrate recognition of the properties of inequalities. However, almost half of the students were unable to solve simple linear inequalities in one unknown.

Q31/M3
Exemplar Item (Solve simple linear inequalities in one unknown) Solve the inequality $-x+3<5$.
Example of Student Work (Could not express the answer in the form $x>a$) $-x<2$.
Example of Student Work (Could not express the answer in inequality) -2

S.3 Measures, Shape and Space Dimension

S.3 students performed steadily in this Dimension. They could find measures in 2-D and 3-D figures, angles related with lines and rectilinear figures. They also performed well in transformation and symmetry, Pythagoras' Theorem, Quadrilaterals and problems related to coordinate geometry. However, more improvement could be shown in items related to definitions and 3-D figures. Comments on students' performances are provided below with examples cited where appropriate (question number *x* /sub-paper *y* quoted as Q*x*/M*y*). More items may also be found in the section ***General Comments***.

Measures in 2-D and 3-D Figures

- Estimation in Measurement: Students' performance was quite good. Most students could choose the method that gave a more accurate reading. They were able to find the range of measures from a measurement of a given degree of accuracy and choose an appropriate unit and the degree of accuracy for real-life measurements. Quite a number of students could estimate measures with justification.

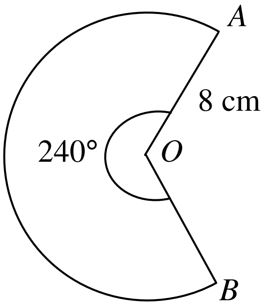
Q47/M1
Example of Student Work (Estimate the area of the notice board – found the sum of the length and width of the board only) $20 \times 5 + 20 \times 3$ $= 160\ cm^2$ //
Example of Student Work (Good performance) The height of the notice board is about 3 drawings, which is about $20 \times 3 = 60cm$ The length of the notice board is about 5 hand span, which is about $20 \times 5 = 100cm$ ∴ The estimated area of the notice board is about $60 \times 100 = 6000cm^2$

- Simple Idea of Areas and Volumes: Students' performance was steady. They did quite well in using the formulas for areas of circles and the surface areas of solids. However, literal presentation and units were often omitted in some students' answers.
- More about Areas and Volumes: Many students could use formulas to calculate areas of sectors and volumes of pyramids. More than half of the students could calculate arc length, surface areas of spheres and distinguish among formulas for lengths, areas, volumes by considering dimensions. Nevertheless, they were not able to use relationships between the sides and volumes of similar figures to solve problems.

Q48/M2

Exemplar Item (Calculate the arc length)

In the figure, the radius of sector OAB is 8 cm and reflex $\angle AOB = 240°$. Find the length of $\overset{\frown}{AB}$. Correct the answer to the nearest 0.1 cm.



Example of Student Work (Mistakenly chose $\angle AOB$)

The length of $\overset{\frown}{AB}$

$2\pi(8) \times \frac{360° - 240°}{360°}$

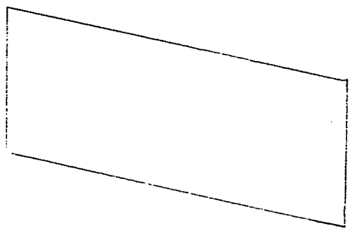
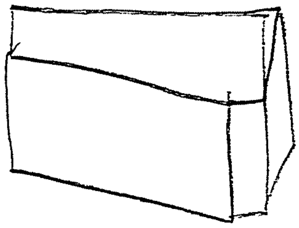
$= 16\pi \times \frac{120°}{360°}$

$= \frac{1920\pi'}{360°}$

$= 16.8$ cm (correct to the nearest 0.1 cm) //

Learning Geometry through an Intuitive Approach

- Introduction to Geometry: Students demonstrated a good recognition of cubes. They could identify 3-D solids from given nets. More than half of the students could sketch a diagram of cylinder and the cross-section of a simple solid, but they performed poorly in recognition of equilateral polygons.

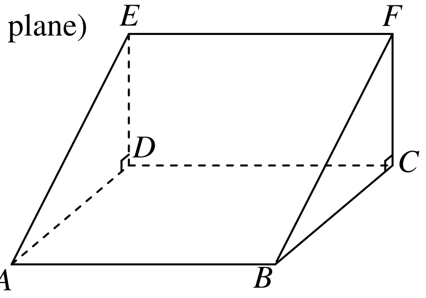
Q34/M4
Example of Student Work (Sketch the cross-section of a cuboid – mistakenly took a parallelogram as the cross section) 
Example of Student Work (Sketch the cross-section of a cuboid – only sketched a diagram of the cuboid that was being cut along the dotted line *AB*) 

- Transformation and Symmetry: Students' performance was good. They demonstrated recognition of basic concepts, but their performance was only satisfactory when they identified the image of a figure after a single transformation.
- Congruence and Similarity: Students could apply the properties of congruent and similar triangles to find the sizes of angles and the lengths of sides in general. They could identify whether 2 triangles are congruent/similar with simple reasons.
- Angles related with Lines and Rectilinear Figures: Students did quite well. They could use the angle properties associated with intersecting lines/parallel lines and the relations between sides and angles of triangles to solve geometric problems.
- More about 3-D figures: Many students could name the axes of rotational symmetries of cubes according to context of the item. They could identify the nets of cubes and matching 3-D objects with various views. Almost half of the students could name the angle between a line and a plane and the projection of an edge on a plane.

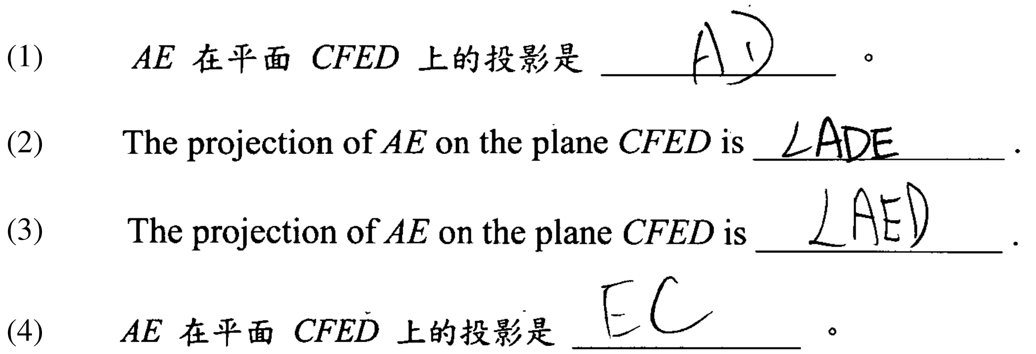
Q38/M2

Exemplar Item (Name the projection of an edge on a plane)

The figure shows a triangular prism. *ABCD* and *CFED* are rectangles. *ABCD* is a horizontal plane and *CFED* is a vertical plane. Name the projection of *AE* on the plane *CFED*.



Example of Student Work (Could not write down the correct projection)



(1) *AE* 在平面 *CFED* 上的投影是 AD 。

(2) The projection of *AE* on the plane *CFED* is ∠ADE .

(3) The projection of *AE* on the plane *CFED* is ∠AED .

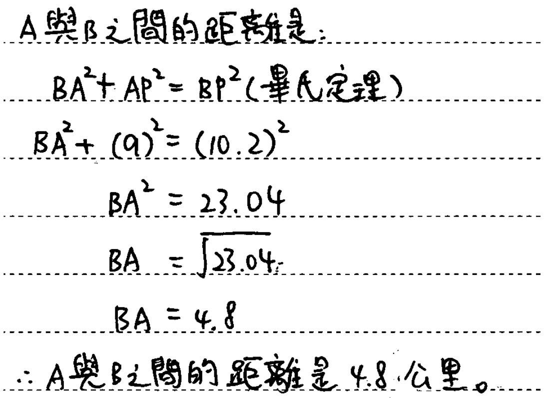
(4) *AE* 在平面 *CFED* 上的投影是 EC 。

Learning Geometry through a Deductive Approach

- Simple Introduction to Deductive Geometry: Quite a number of students could identify the correct geometric proof. Besides, more than half of the students were able to identify perpendicular bisectors of a triangle.
- Pythagoras' Theorem: Students did well. They could use Pythagoras' Theorem to solve simple problems and determine whether the given triangles were right-angled triangles or not by using the converse of Pythagoras' Theorem.

Q49/M3

Example of Student Work (Good performance)



A與B之間的距離是：

$BA^2 + AP^2 = BP^2$ (畢氏定理)

$BA^2 + (9)^2 = (10.2)^2$

$BA^2 = 23.04$

$BA = \sqrt{23.04}$

$BA = 4.8$

∴ A與B之間的距離是 4.8 公里。

- Quadrilaterals: Students performed well. They could use the properties of rhombuses and trapeziums in numerical calculations.

Learning Geometry through an Analytic Approach

- Introduction to Coordinates: More than half of the students could match a point with its image under rotation with the origin through 180° or reflection with respect to *y*-axis in the coordinate plane. Besides, many students could calculate areas of simple figures.

Q49/M4
Example of Student Work (Calculate the area of figure – Mistakenly wrote the unit as cm^2)
Length of BC, AD and DC are $3-(-4)=7$, $3-(-2)=5$, and $4-1=3$ respectively. ∴ Area of trapezium ABCD $=\frac{(7+5)(3)}{2}$ $=18$ cm^2
Example of Student Work (Correct solution)
AD $= 3-(-2) = 5$ BC $= 3-(-4) = 7$ DC $= 4-1 = 3$ ABCD面積 : $\frac{(5+7)\times 3}{2}$ $= 18$ 平方單位

- Coordinate Geometry of Straight Lines: Students performed well. They could find the slopes of straight lines, use the distance formula and the mid-point formula. They also demonstrated recognition of the conditions for perpendicular lines.

Trigonometry

- Trigonometric Ratios and Using Trigonometry: Students demonstrated recognition of the ideas of sine, cosine and tangent ratios. They could solve simple 2-D problems involving one right-angled triangle, but their performance was only satisfactory in recognition of the idea of bearing.

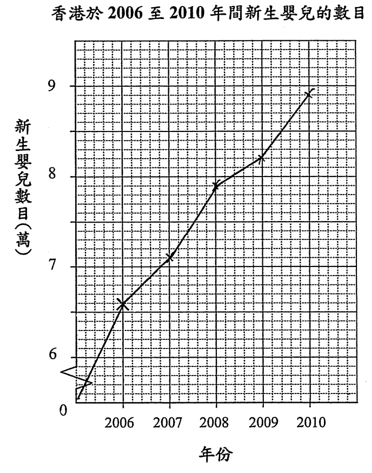
Q50/M1
Example of Student Work (Find the angle of elevation – Poor presentation) $\tan\theta = \frac{150}{85} = 60.5$ (準確至三位有效數字) ∴由B測得塔頂C的仰角為 $60.5°$
Example of Student Work (Correct solution) Let angle B be the θ ∵ $\tan\theta = \frac{150}{85}$ ∴ $\theta = 60.5°$ ∴ the angle of elevation of the top C of the tower from B IS $60.5°$

S.3 Data Handling Dimension

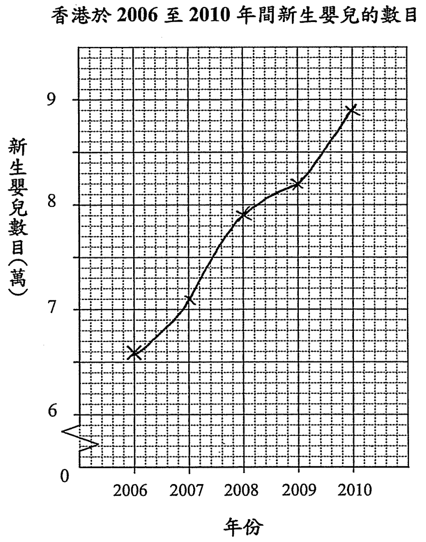
The performance of S.3 students was quite good in this Dimension. They did well in items related to organizing the same set of data by different grouping methods, constructing simple statistical charts and interpretation of information, reading information from the graphs and calculating the empirical probability. However, performance was weak when students were asked to distinguish discrete and continuous data and find the mean from a set of grouped data. Comments on students' performance are provided below with examples cited where appropriate (question number *x* / sub-paper *y* quoted as Q*x*/M*y*). More examples may also be found in the section ***General Comments***.

Organization and Representation of Data

- Introduction to Various Stages of Statistics: Students' performance was steady. Many students not only used simple methods to collect and organize data, but also could organize the same set of data by different grouping methods. Nevertheless, students didn't know how to distinguish discrete and continuous data.
- Construction and Interpretation of Simple Diagrams and Graphs: Students could construct broken line graphs to represent a set of data and interpret simple statistical charts. They were also able to read information from the diagrams/graphs and identify sources of deception in misleading graphs/accompanying statements. However, more than half of the students could not choose appropriate diagrams/graphs to present a set of data.

Q43/M2
Example of Student Work (Construction of broken line graphs – Mistakenly extended one end of the broken line graph) 

Example of Student Work (Construction of broken line graphs – Mistakenly connected all the points by curve)



Analysis and Interpretation of data

- Measures of Central Tendency: Many students could find the mean and median from ungrouped data and identify sources of deception in cases of misuse of averages. From a set of grouped data, they fared better in finding the modal class than in finding the mean. Besides, more than half of the students could not calculate the weighted mean of a set of data.

Q41/M4

Exemplar Item (Find the mean from a set of grouped data)

The following table shows the number of times 40 members practised in a yoga centre last month.

Number of times	0 – 4	5 – 9	10 – 14	15 – 19	20 – 24
Number of members	2	12	16	8	2

Find the mean number of times 40 members practised in the yoga centre last month.

Q41/M3

Example of Student Work (Calculate the weighted mean of a set of data)

Jackson participated in a gymnastics competition of a university. The following table shows the weight of each marking item and the marks that he got in these items.

	Marking item		
	Skill	Artistry	Difficulty
Weight	2	1	1
Mark	7	4	3

Find the weighted mean mark of Jackson.

Probability

- Simple Idea of Probability: Students fared better in finding the empirical probability than in finding the theoretical probability by listing.

Q42/M1

Exemplar Item (Calculate the empirical probability)

There are 1 blue pen, 1 red pen and 1 green pen inside the pencil case of Sally. There are 1 blue pen and 1 red pen inside the pencil case of Candy. If a pen is drawn from **EACH** pencil case at random, find the probability of getting 2 blue pens.

Example of Student Work (Considered the number of blue pens and the total number of pens only)

$\frac{2}{5}$

General Comments on S.3 Student Performances

The overall performance of S.3 students was satisfactory. They did quite well in Number and Algebra Dimension and Data Handling Dimension. Performance was steady in Measures, Shape and Space Dimension.

The areas in which students demonstrated adequate skills are listed below:

Directed Numbers and the Number Line:

- Demonstrate recognition of the ordering of integers on the number line (e.g. Q21/M3).
- Add, subtract, multiply and divide directed numbers (e.g. Q22/M2).

Numerical Estimation:

- Determine whether to estimate or to compute the exact value in a simple context (e.g. Q1/M3).

Approximation and Errors:

- Round off a number to a certain number of significant figures (e.g. Q1/M4).

Rational and Irrational Numbers:

- Demonstrate recognition of the integral part of $\sqrt{a}$, where a is a positive integer not greater than 200 (e.g. Q4/M2).

Rate and Ratio:

- Use rate and ratio to solve simple real-life problems (e.g. Q24/M3).

Formulating Problems with Algebraic Language:

- Translate word phrases/contexts into algebraic languages (e.g. Q4/M4).

Formulas:

- Substitute values of formulas and find the value of a specified variable (e.g. Q30/M1).

Estimation in Measurement:

- Find the range of measures from a measurement of a given degree of accuracy (e.g. Q9/M1).
- Choose an appropriate unit and the degree of accuracy for real-life measurements (e.g. Q10/M3).
- Reduce errors in measurements (e.g. Q10/M4).

Introduction to Geometry:

- Identify types of angles with respect to their sizes (e.g. Q11/M4).

Transformation and Symmetry:

- Name the single transformation involved in comparing the object and its image (e.g. Q12/M1 and Q13/M3).

Congruence and Similarity:

- Demonstrate recognition of the properties of congruent and similar triangles (e.g. Q35/M4).

Angles related with Lines and Rectilinear Figures:

- Demonstrate recognition of interior angles of polygons (e.g. Q16/M2).

Introduction to Coordinates:

- Use an ordered pair to describe the position of a point in the rectangular coordinate plane and locate a point of given rectangular coordinates (e.g. Q39/M4).

Introduction to Various Stages of Statistics:

- Organize the same set of data by different grouping methods (e.g. Q50/M2).

Construction and Interpretation of Simple Diagrams and Graphs

- Interpret simple statistical charts including stem-and-leaf diagrams, pie charts, histograms, scatter diagrams, broken line graphs, frequency polygons and curves, cumulative frequency polygons and curves (e.g. Q51/M3).

Measures of Central Tendency:

- Find the mean, median and mode from a set of ungrouped data (e.g. Q41/M2).

Other than items in which students performed well, the Assessment data also provided some entry points to strengthen teaching and learning. Items worthy of attention are discussed below:

Manipulations of Simple Polynomials

- Distinguish polynomials from algebraic expressions (e.g. Q6/M3): Almost half of the students could distinguish polynomials from algebraic expressions and so they chose the correct answer D. Each of the remaining options was chosen by over 10% of students.

Q6/M3
Which of the following is a polynomial? A. $\frac{x^2}{y}+1$ B. 2^x+y+1 C. $x^2+\sqrt{y}+1$ D. x^2+y+1

- Add or subtract polynomials of at most 4 terms (e.g. Q5/M4): The performance of students was weak in dealing with the addition or subtraction of like terms / unlike terms. Only half of the students chose the correct answer B. Some students chose option A instead.

Q5/M4
Simplify $(4a-2ab)-(2ab+3a)$. A. $7a-4ab$ B. $a-4ab$ C. $7a$ D. a

- Demonstrate understanding of the meaning of roots of equations (e.g. Q7/M4): The design of the item is: students are expected to find out which equation with the root 12 by the method of substitution instead of solving each equation one by one. The result showed that only half of the students were able to choose the correct answer D. Some students chose option B.

Q7/M4

Which of the following is an equation with the root 12?

A. $\frac{x}{6}+4=\frac{x}{2}+2$

B. $\frac{x}{4}+3=\frac{x}{3}+3$

C. $\frac{x}{3}+2=\frac{x}{4}+4$

D. $\frac{x}{2}+1=\frac{x}{6}+5$

Identities

- Tell whether an equality is an equation or an identity (e.g. Q8/M4): Quite a number of students chose the correct answer C. However, there were over 10% of students who chose option A.

Q8/M4

Which of the following is an identity?

A. $2x-9=(x-3)^2$

B. $2x-9=9-2x$

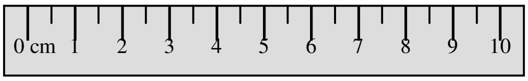
C. $2x-9=(x-3)+(x-6)$

D. $2x-9=2(x-9)$

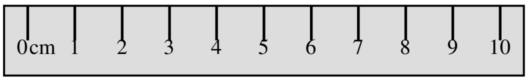
Estimation in Measurement

- Choose an appropriate measuring tool and technique for real-life measurements (e.g. Q10/M2): Some students chose option A and this result reflected that students could choose appropriate measuring tools in general but they neglected to make use of suitable technique.

Q10/M2



Ruler *A*



Ruler *B*

The above figure shows ruler *A* and ruler *B* with different graduations. Thomas wants to find the thickness of a ten-cent coin. Of the following methods, which one is the best?

A. Thomas measures the thickness of a ten-cent coin using ruler *A*.

B. Thomas measures the thickness of a ten-cent coin using ruler *B*.

C. Thomas measures the thickness of 50 ten-cent coins using ruler *A* and then divides the thickness by 50.

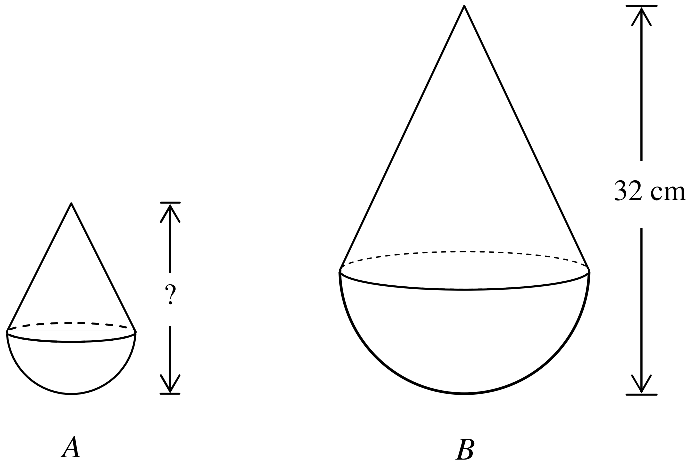
D. Thomas measures the thickness of 50 ten-cent coins using ruler *B* and then divides the thickness by 50.

More about Areas and Volumes

- Use the relationships between sides and surface areas/volumes of similar figures to solve related problems (e.g. Q11/M1): Almost half of the students treated the ratio between the heights of two similar figures the same as the ratio between their volumes and so they chose option C.

Q11/M1

In the figure, *A* and *B* are two similar solids. The volumes of *A* and *B* are V cm^3 and $8V$ cm^3 respectively. If the height of *B* is 32 cm, find the height of *A*.



A. 16 cm

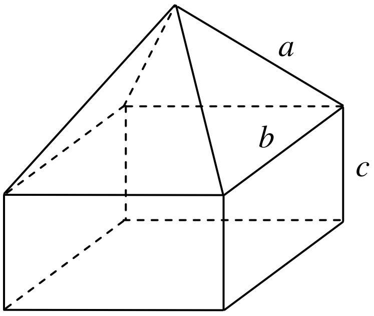
B. 8 cm

C. 4 cm

D. 2 cm

- Distinguish among formulas for lengths, areas and volumes by considering dimensions (e.g. Q11/M3): Almost half of the students chose the correct answer B. Some students chose option C.

Q11/M3



The solid in the figure is formed by a pyramid and a cuboid. The base of the pyramid is a square of side *b*. The length of slant edge is *a* and the height of cuboid is *c*. Which of the following could be expressed by $b\left(\sqrt{3}a+b+4c\right)$?

A. Volume of the solid
B. Total surface area of the solid
C. Total sum of lengths of the solid
D. Height of the solid

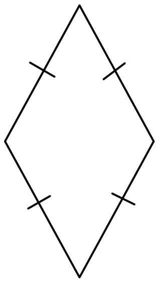
Introduction to Geometry

- Determine whether a polygon is regular, convex, concave, equilateral or equiangular (e.g. Q33/M4): Quite a number of students chose figures P and R. They could not identify that polygon Q was indeed an equilateral.

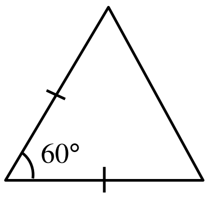
Q33/M4

Which of the following polygons **MUST** be equilateral? (May be more than one answer)

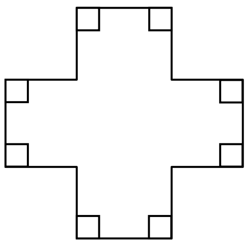
P.



Q.



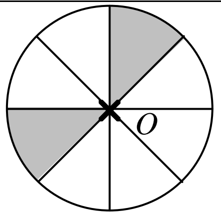
R.



Transformation and Symmetry

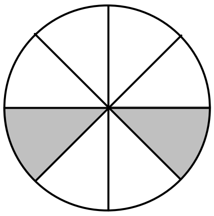
- Identify the image of a figure after a single transformation (e.g. Q13/M4): Some students chose option C. They confused the clockwise direction with the anticlockwise direction.

Q13/M4

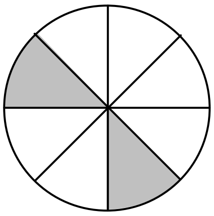


Find the image of the above figure after rotating about O through 90° in anticlockwise direction.

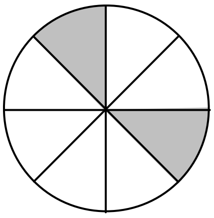
A.



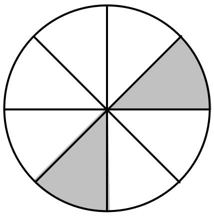
B.



C.

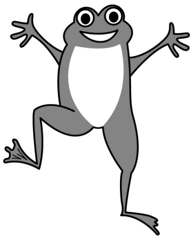


D.



- Demonstrate recognition of the effect on the size and shape of a figure under a single transformation (e.g. Q14/M3): Almost half of the students chose option B. They mistakenly thought that the shape of the figure would be changed after reflection.

Q14/M3



Will the size and shape of the above figure be changed after reflection?

	Size	Shape
A.	unchanged	unchanged
B.	unchanged	changed
C.	changed	unchanged
D.	changed	changed

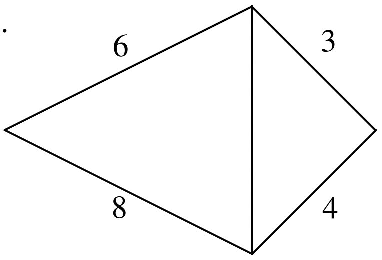
Congruence and Similarity

- Demonstrate recognition of the conditions for congruent and similar triangles (e.g. Q15/M2): Half of the students chose the correct answer D only. Some students chose option C. They mistakenly took the differences between the lengths of the corresponding sides to be the condition to judge whether two triangles were similar.

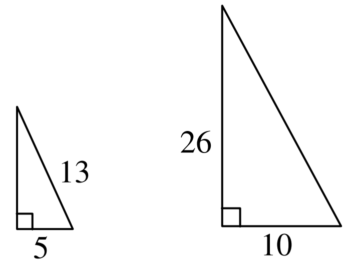
Q15/M2

Which of the following figures shows two similar triangles?

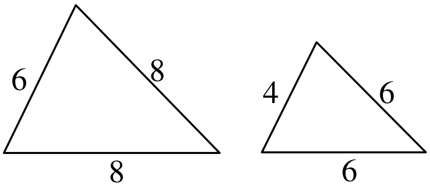
A.



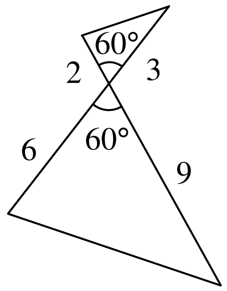
B.



C.



D.



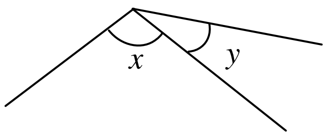
Angles related with Lines and Rectilinear Figures

- Demonstrate recognition of the terminologies on angles with respect to their positions relative to lines and polygons (e.g. Q15/M4): Almost half of the students chose the correct answer A. Some students chose option C.

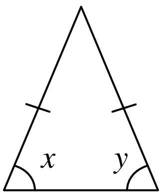
Q15/M4

In which of the following figures, are x and y adjacent angles?

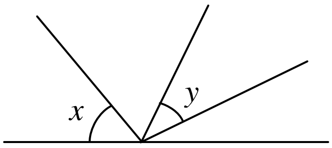
A.



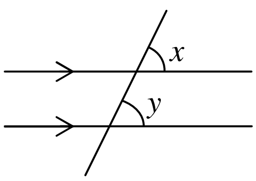
B.



C.



D.



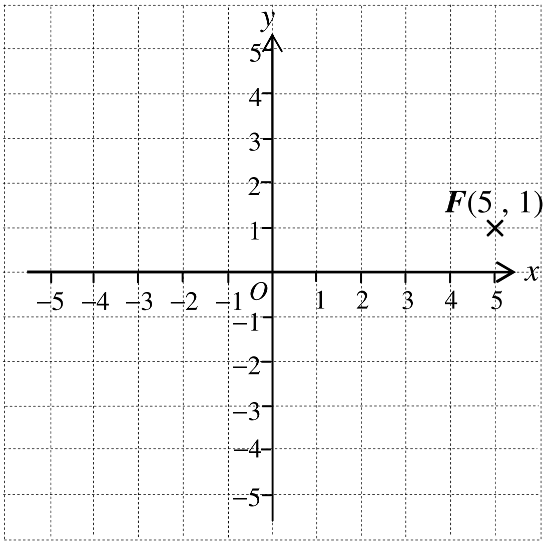
Introduction to Coordinates

- Match a point under a single transformation with its image in the rectangular coordinate plane (e.g. Q17/M4): Students' performance was only fair with items related to the single transformation including rotation. Some students chose option B. As in the last year, they confused rotation with reflection.

Q17/M4

In the figure, **F**(5 , 1) is rotated about the origin O through 180° to **F'**. The coordinates of **F'** are

A. (5 , −1).
B. (−5 , 1).
C. (−5 , −1).
D. (−1 , −5).



Introduction to Various Stages of Statistics

- Distinguish discrete and continuous data (e.g. Q20/M1): Only some students chose the correct answer A. Almost half of the students mistakenly assumed that class numbers were continuous data and the weight of students were discrete data and so they chose the option C.

Q20/M1

Determine whether each of the following data is discrete or continuous.

(i) The class numbers of 3B students
(ii) The weights of 3B students

	(i)	(ii)
A.	Discrete data	Continuous data
B.	Discrete data	Discrete data
C.	Continuous data	Discrete data
D.	Continuous data	Continuous data

Construction and Interpretation of Simple Diagrams and Graphs

- Choose appropriate diagrams/graphs to present a set of data (e.g. Q20/M2): Some students chose the option C. They mistakenly represented the data by broken line graph.

Q20/M2

After organizing the data of the blood pressure of 60 patients, a nurse has constructed a frequency distribution table as below.

Blood pressure (mmHg)	110 – 119	120 – 129	130 – 139	140 – 149	150 – 159	160 – 169
Frequency	3	14	20	12	7	4

Which of the following is suitable to present the data in the above table?

A. Histogram
B. Scatter diagram
C. Broken line graph
D. Stem-and-leaf diagram

Best performance of S.3 Students in TSA 2012

Students sitting for each sub-paper were ranked according to their scores and the performances of approximately the top 10% were singled out for further analysis. The performances of these students are described below.

Among these students, the majority of them achieved a full score or lost at most five score points in the whole assessment. They demonstrated almost complete mastery of the concepts and skills assessed by the sub-papers they attempted.

Most of these students were able to formulate linear equations in one unknown from simple contexts (e.g. Q5/M1), reduce errors in measurements (e.g. Q10/M1), substitute values of formulas and find the value of a specified variable (e.g. Q30/M1), use rate and ratio to solve simple real-life problems (e.g. Q45/M1 and Q24/M3), calculate volumes of pyramids, circular cones and spheres (e.g. Q49/M1), interpret simple statistical charts (e.g. Q51/M1 and Q51/M3), demonstrate recognition of the integral part of $\sqrt{a}$, (e.g. Q4/M2), translate word phrases/contexts into algebraic languages (e.g. Q5/M2), add, subtract, multiply and divide directed numbers (e.g. Q22/M2), find the sine, cosine and tangent ratios for angles between 0° to 90° and vice versa (e.g. Q39/M2), solve simple problems on growths and depreciations (e.g. Q45/M2), perform change of subject in simple formulas but not including radical sign (e.g. Q46/M2), use the formulas for circumferences and areas of circles (e.g. Q47/M2), solve simple selling problems (e.g. Q43/M3), use the laws of integral indices to simplify simple algebraic expressions (e.g. Q44/M3), solve a system of simple linear simultaneous equations by algebraic methods (e.g. Q46/M3), name the single transformation involved in comparing the object and its image (e.g. Q12/M4), use an ordered pair to describe the position of a point in the rectangular coordinate plane and locate a point of given rectangular coordinates (e.g. Q39/M4).

The examples of work by these students are illustrated:

Students with the best performance could set up and solve the problem correctly with a complete solution.

Q45/M1

Example of Student Work (Find the actual length of the plane)

Let x be the actual length of the plane

$\frac{5.4}{x} = \frac{1}{1500}$

$5.4(1500) = x$

$x = 8100$

∴ The actual length of the plane is 81 m.

Q47/M2

Example of Student Work (Find the radius and the circumference of the circle)

(a) r的值是：

$\pi r^2 = 25\pi$

$r = 5$

∴ r的值是5，圓形的半徑為5cm

(b) 該圓形的圓周為：

$2 \times \pi \times 5$

$= 10\pi$ cm

Students with the best performance could make good use of the given conditions and solve the problem systematically.

Q46/M4

Example of Student Work (Using the result of (a) to find *C* in part (b))

(a) $F = \frac{9C}{5} + 32$

$F - 32 = \frac{9C}{5}$

$5F - 160 = 9C$

$C = \frac{5(F-32)}{9}$

(b) $C = \frac{5(F-32)}{9}$

$C = \frac{5(104-32)}{9}$

$C = \frac{360}{9}$

$C = 40$

Students with the best performance could show steps clearly and used correct reasoning to set up the conclusion.

Q47/M1

Example of Student Work (Estimate the area of the notice board)

The hand span of the boy $\approx$ the length of a drawing

$\therefore$ The length of a drawing $\approx$ 20 cm

The width of the notice board has 3 drawings

$\therefore$ The width of the board $\approx 3 \times 20 \approx 60$ cm

The length of the notice board included nearly 5 drawings

$\therefore$ The length of the board $\approx 5 \times 20 \approx 100$ cm

$\therefore$ The area of the board $\approx 60 \times 100 \approx 6000$ cm^2

Q51/M2

Example of Student Work (Estimate the bus fares needed for Susan and her brother)

$9.7 = 10　姊姊五天共需車費：5×10

$4.9 = 5　= 50元

弟弟五天共需車費 = 5×5

= 25元

我所用的是上捨入法，把數目加爲整數，不會減小。

Some common weaknesses of high-achieving students are listed as follows:

- Some students could not distinguish discrete and continuous data.
- Some students could not demonstrate recognition of variables of polynomials.
- Some students could not plot the graph of the equation $y = 4$.
- Some students could not demonstrate recognition of adjacent angles.

Comparison of Student Performances in Mathematics at Secondary 3 TSA 2010, 2011 and 2012

This was the seventh year that Secondary 3 students participated in the Territory-wide System Assessment. The percentage of students achieving Basic Competency this year was 79.8% which was about the same as last year.

The percentages of students achieving Basic Competency from 2010 to 2012 are listed below:

Table 8.5 Percentages of S.3 Students Achieving Mathematics Basic Competency from 2010 to 2012

Year	% of Students Achieving Mathematics Basic Competency
2010	80.1
2011	80.1
2012	79.8

The performances of S.3 students over the past three years in each Dimension of Mathematics are summarized below:

Number and Algebra Dimension

- Directed Numbers and the Number Line: The performance of students has been good over the past three years.
- Numerical Estimation: Students were able to determine whether to estimate or to compute the exact value in a simple context. The performance was better this year when students were asked to estimate values with reasonable justifications.
- Approximation and Errors: Performance was steady on conversion of significant figures. Students were more capable of representing a large number in scientific notation this year.
- Rational and Irrational Numbers: Performance remained good. The majority of students could represent real numbers on the number line.
- Using Percentages: Students could solve simple selling problems. Performance was steady in solving problems on growths and depreciations. However, there is room for improvement on problems related with simple interest. They could not master the concept of percentage (e.g. they confused y and y %).

- Rate and Ratio: Students did quite well in the calculation and recognition of rate and ratio. As in the last year, the presentation of students' answers remained unclear and incomplete in solving simple real-life problems.
- Formulating Problems with Algebraic Language: Students did quite well in translating word phrases/contexts into algebraic languages. The performance remained good in items involved numerical calculation. When the problems involved the n^{th} term of number sequence, there was room for improvement in students' performance.
- Manipulations of Simple Polynomials: Students showed improvement in distinguishing polynomials from algebraic expressions this year. They performed steadily in addition, subtraction or multiplication of polynomials, but their performance was only fair in recognition of terminologies.
- Laws of Integral Indices: Performance was steady in using the laws of integral indices to simplify simple algebraic expressions.
- Factorization of Simple Polynomials: Performance was only fair in recognition of factorization as a reverse process of expansion. Students performed steadily in factorization of simple polynomials by taking out common factors, using the difference of two squares, the perfect square expressions or the cross method.
- Linear Equations in One Unknown: Students showed improvement in formulating linear equations in one unknown from simple contexts. They performed steadily in solving simple equations.
- Linear Equations in Two Unknowns: They did quite well in plotting graphs of linear equations $ax + by + c = 0$, but they were still weak in plotting the graphs of $my + n = 0$. They could recognize that the graphs of equations of the form $ax + by + c = 0$ are straight lines. Besides, students generally were able to solve simultaneous equations by algebraic methods or graphical method.
- Identities: Performance of students was still fair in recognition of identity. Performance was steady when they were asked to expand simple algebraic expressions by using the difference of two squares and the perfect square expressions.
- Formulas: Performance of students was only fair in manipulating algebraic fractions. They were capable of solving problems with numerical calculation.

- Linear Inequalities in One Unknown: Students showed improvement in recognition of the properties of inequalities. However, their performance was only fair in solving simple linear inequalities in one unknown.

Measures, Shape and Space Dimension

- Estimation in Measurement: Students could choose appropriate methods to reduce errors in measurements. The performance was similar to last year in items relating to choosing an appropriate measuring tool and technique and estimating measures with justification. They showed improvement in choosing an appropriate unit and the degree of accuracy.
- Simple Idea of Areas and Volumes: Students performed steadily in calculation involving radius, circumference and area of circle. They did quite well in applying the formulas for surface areas of simple solids.
- More about Areas and Volumes: Performance was quite good in calculation involving areas of sectors, surface areas and volumes. Nonetheless, their performance remained unsatisfactory in items dealing with relationships of sides and volumes in similar figures and distinguishing among formulas for lengths, areas, volumes by considering dimensions.
- Introduction to Geometry: Students could identify the common terms in geometry and the relationship between 3-D solids and their nets. They performed steadily in sketching simple solids and cross-sections of simple solids.
- Transformation and Symmetry: Performances of students were quite good in general, although they performed fairly on the problems involving rotation.
- Congruence and Similarity: Students could use the properties of congruent and similar triangles to solve triangles. However, they could not demonstrate recognition of the conditions for congruent and similar triangles.
- Angles related to Lines and Rectilinear Figures: The performance of students was good. They could use the angle properties associated with intersecting lines/parallel lines and the properties of triangles to solve simple geometric problems.
- More about 3-D Figures: Students' performance was still good in matching 3-D objects from 2-D representations. Performance improved in dealing with the angle between a line and a plane and the angle between 2 planes.

- Simple Introduction to Deductive Geometry: Performance was steady. Students generally could not identify which of the given proofs was incorrect.
- Pythagoras' Theorem: Performance was quite good. Students were capable of applying Pythagoras' Theorem to solve simple problems.
- Quadrilaterals: Performance remained steady. Students could use the properties of quadrilaterals to solve geometric problems.
- Introduction to Coordinates: They demonstrated good recognition of the coordinates system. They showed improvement when they were asked to calculate area of simple figure. However, they were still weak in items relating to transformation.
- Coordinate Geometry of Straight Lines: Students were more capable in using the distance formula. Performance in applying the formula of slope and mid-point formula improved.
- Trigonometric Ratios and Using Trigonometry: Students showed the recognition of basic concepts. They could solve simple 2-D problems involving one right-angled triangle. However, there was room for improvement in the recognition of the idea of bearing.

Data Handling Dimension

- Introduction to Various Stages of Statistics: Performance was steady. Students were still weak in distinguishing between discrete and continuous data.
- Construction and Interpretation of Simple Diagrams and Graphs: Performance was steady in constructing simple statistical charts. They showed improvement in interpreting simple statistical charts, reading information from diagrams/graphs and identifying sources of deception in misleading graphs.
- Measures of Central Tendency: The items related to finding mean from a set of grouped data or calculating the weighted mean were still the weak area for students, but students performed quite well in other items.
- Simple Idea of Probability: They performed more than satisfactorily in calculating the empirical probability this year. However, their performance was only fair in calculating the theoretical probability by listing.