Results of Secondary 3 Mathematics in Territory-wide System Assessment 2019

The percentage of Secondary 3 students achieving Mathematics Basic Competency in 2019 is 79.6%.

Secondary 3 Assessment Design

The design of assessment tasks for S.3 was based on the documents *Mathematics Curriculum: Basic Competency for Key Stage 3 (Tryout Version)* and *Syllabuses for Secondary Schools – Mathematics (Secondary 1 – 5), 1999*. The tasks covered the three dimensions of the mathematics curriculum, namely **Number and Algebra**, **Measures**, **Shape and Space**, and **Data Handling**. They focused on the Foundation Part of the S1 – 3 syllabuses in testing the relevant concepts, knowledge, skills and applications.

The Assessment consisted of various item types including multiple-choice questions, fill in the blanks, answers-only questions and questions involving working steps. The item types varied according to the contexts of the questions. Some test items consisted of sub-items. Besides finding the correct answers, students were also tested in their ability to present solutions to problems. This included writing out the necessary statements, mathematical expressions and explanations.

The Assessment consisted of 146 test items (198 score points), covering all of the 129 Basic Competency Descriptors. These items were organized into four sub-papers, each 65 minutes in duration and covering all three dimensions. Some items appeared in more than one sub-paper to act as inter-paper links and to enable the equating of test scores. Each student was required to attempt one sub-paper only. The number of items on the various sub-papers is summarized in Table 8.7. These numbers include several overlapping items.

Table 8.7 Number of Items and Score Points for S.3

Subject	No. of Items (Score Points)				
	Paper 1	Paper 2	Paper 3	Paper 4	Total*
Mathematics					
Written Paper					
Number and Algebra	22 (31)	22 (31)	20 (26)	21 (26)	64 (84)
Measures, Shape and Space	19 (24)	19 (24)	20 (27)	20 (28)	64 (84)
Data Handling	6 (10)	6 (10)	7 (12)	6 (11)	18 (30)
Total	47 (65)	47 (65)	47 (65)	47 (65)	146 (198)

* Items that appear in different sub-papers are counted once only.

The item types of the sub-papers were as follows:

Table 8.8 Item Types of the Sub-papers

Section	Percentage of Score Points	Item Types
A	~ 30%	• Multiple-choice questions: choose the best answer from among four options
B	~ 30%	• Calculate numerical values • Give brief answers
C	~ 40%	• Solve application problems showing working steps • Draw diagrams or graphs • Open-ended questions requiring reasons or explanations

Performance of Secondary 3 Students Achieving Basic Competency in 2019

Secondary 3 Number and Algebra Dimension

S.3 students performed steadily in this dimension. The majority of students demonstrated recognition of directed numbers, formulating problems with algebraic language and linear inequalities in one unknown. Performance was only fair in items related to numerical estimation, using percentages and identities. Comments on students' performances are provided with examples cited where appropriate (question number *x* / sub-paper *y* quoted as Q*x*/M*y*). More examples may also be found in the section ***General Comments***.

Number and Number Systems

- Directed Numbers and the Number Line: Students were in general able to use directed numbers to represent the remaining stored value and the overdraft on an Octopus card. The majority of students could demonstrate recognition of the ordering of integers on the number line and they were quite good in the basic operations of directed numbers.

- Numerical Estimation: Many students were able to determine whether the value mentioned in a simple context was obtained by estimation or by computation of the exact value. Their performace in judging the reasonability of answers obtained was good. Nevertheless, many students were not able to estimate the values with reasonable justifications according to the question. Their performance was not satisfactory.

Q45/M4
Exemplar Item (Estimate the total amount needed for buying the items and judge whether Miss Chan can get the discount) In a department store, customers can get a discount for any purchase of $500 or above. Miss Chan bought 3 items in the store. The prices of the items are $256, $102 and $201 respectively. Based on the description above, give **an appropriate approximation** for the price of each of the items. Hence, estimate the total amount that Miss Chan paid for the items. Briefly explain whether she can get the discount.

Example of Student Work (Without giving approximations for the price of each of the items)

The three items are priced $256, $102 & $201 respectively.
The total amount she bought = $256 + $102 + $201
= $559
> $500

∴ Miss Chan * can / cannot get the discount. (*Circle the correct answer)

Example of Student Work (Used wrong method to find the approximation)

The price she need to pay
≈ 260 + 100 + 200
≈ $560
∵ $560 > $500, which is the price she need to purchase in order to get the discount.

∴ Miss Chan * can / cannot get the discount. (*Circle the correct answer)

Example of Student Work (Good performance)

The total amount = 256 + 102 + 201
≈ 250 + 100 + 200
≈ $550
∵ Her amount $550 > $500

∴ Miss Chan * can / cannot get the discount. (*Circle the correct answer)

- Approximation and Errors: The majority of students could round a decimal greater than 1 to 2 decimal places, but half of the students were not capable of rounding a decimal less than 1 to 3 significant figures. In addition, most students were able to convert a number in scientific notation to decimal.

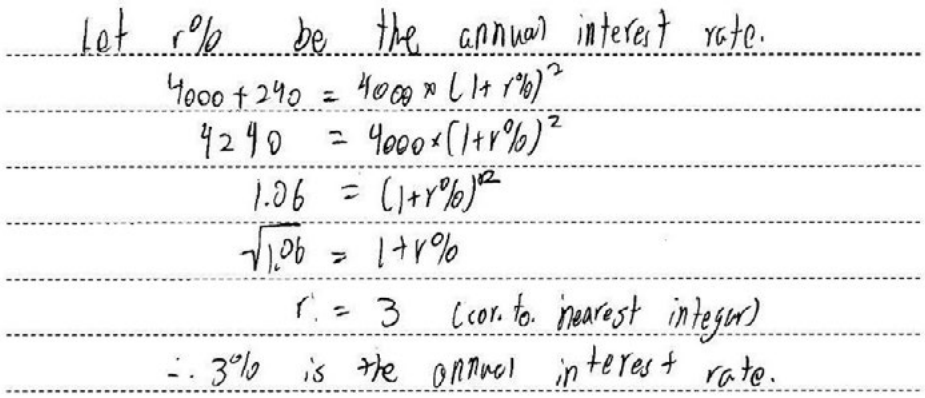
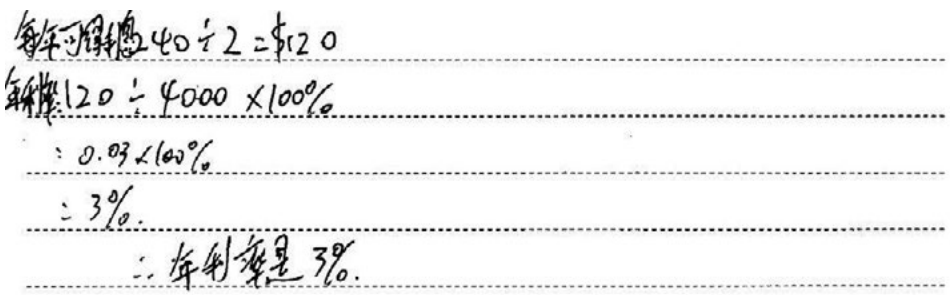
Q22/M2
Exemplar Item (Round off a decimal to 3 significant figures.) Round off 0.018 71 to 3 significant figures.
Example of Student Work (Not able to round off a decimal to 3 significant figures) (1) 0.02 (2) 0.019 (3) 0.0290 (4) 0.187

- Rational and Irrational Numbers: Many students were able to represent a fraction on a number line. They were able to demonstrate recognition of the integral part of $\sqrt{a}$.

Comparing Quantities

- Using Percentages: The performance of students was fair in solving problems regarding depreciations and compound interest. There was room for improvement in solving problems on simple interest to find the interest rate.

Q40/M2
Exemplar Item (Find the selling price) The cost of a smartphone is $6 500. The shop sells it at a loss of 12%. Find the selling price of the phone.
Example of Student Work (Confused loss with profit per cent) 智能電話售價： $6500 × (1+12%) =$7280
Example of Student Work (Correct solution) 6500 × (1−12%) =$5720 ∴該部智能電話的售價是$5720。

Q40/M1
Exemplar Item (Find the annual interest rate) Morris deposits \$4 000 in a bank. After 2 years, he will receive a **simple interest** of \$240. Find the annual interest rate.
Example of Student Work (Confused simple interest with compound interest) 
Example of Student Work (Correct solution) 

- Rate and Ratio: The majority of students were able to use rate and ratio to solve simple real-life problems and demonstrate recognition of the difference between rate and ratio.

Observing Patterns and Expressing Generality

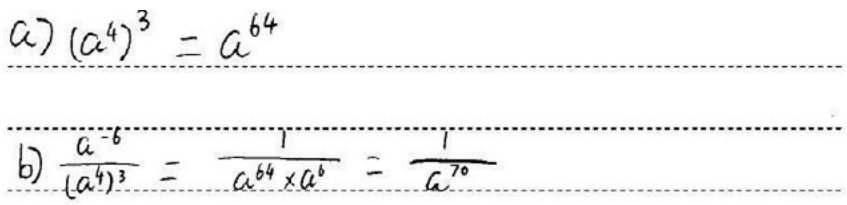
- Formulating Problems with Algebraic Language: Students did well in formulating simple inequalities from simple contexts, substituting values into some common formulas and finding the value of a specified variable, and describing patterns by writing the next few terms in arithmetic sequences from several consecutive terms of integral values. They could translate contexts into algebraic languages. However, only almost half of the students were able to distinguish the difference between $(-2)^n$ and -2^n.
- Manipulations of Simple Polynomials: Students did quite well in dealing with the additions, subtractions and expansions of simple polynomials, but almost half of the

students could not distinguish polynomials from algebraic expressions. They could not demonstrate good recognition of terminologies.

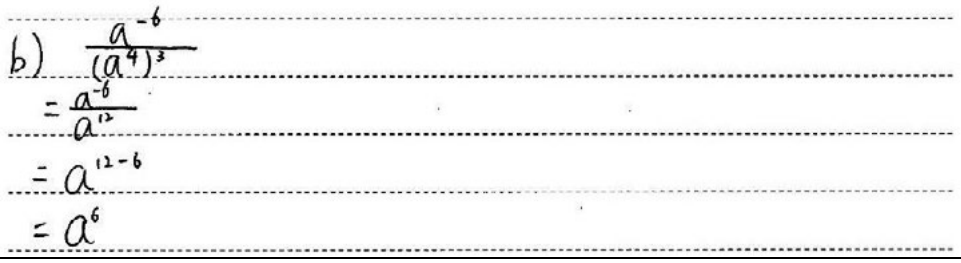
- Laws of Integral Indices: The majority of students were able to find the value of a^n, but half of the students were not able to understand the concept of $x^{-n} = \frac{1}{x^n}$ where *a* and *n* are integers.

Q41/M2

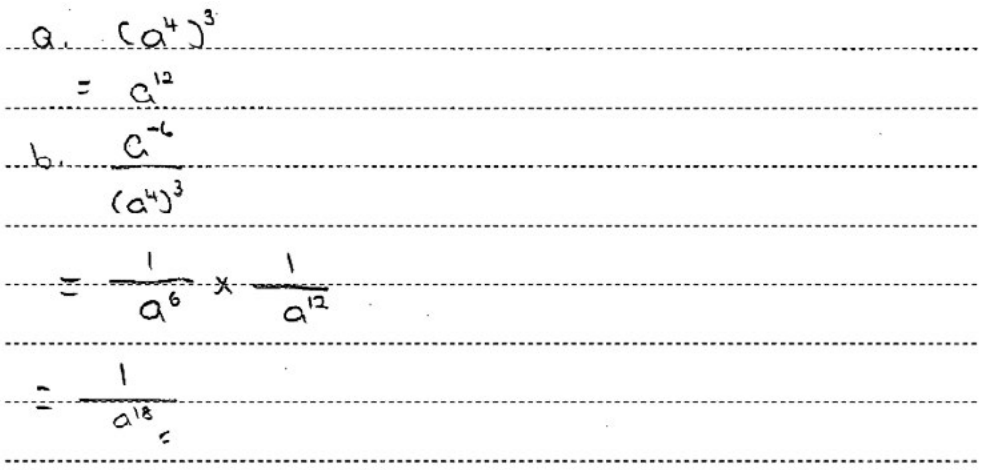
Example of Student Work (Has mistakenly taken $(x^m)^n = x^{m^n}$ in part (a))



Example of Student Work (Has mistakenly taken $\frac{1}{x^m} = x^m$ in part (b))



Example of Student Work (Correct solution)

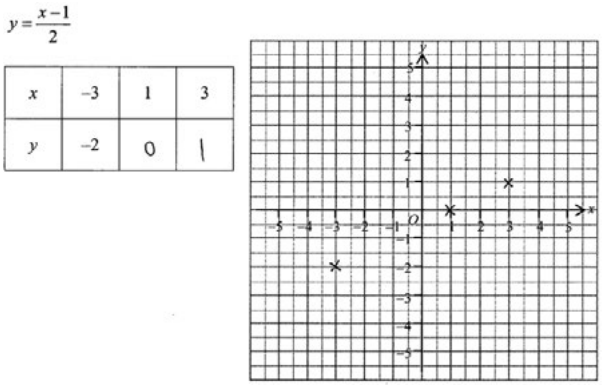
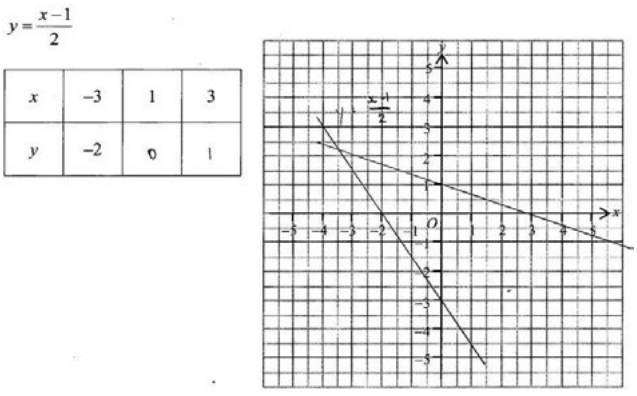


- Factorization of Simple Polynomials: Students were able to demonstrate recognition of factorization as a reverse process of expansion. They did quite well in factorizing simple polynomials by using the difference of two squares, perfect square expressions, cross method and grouping terms.

Algebraic Relations and Functions

- Linear Equations in One Unknown: The performance of students was quite good in solving simple equations. Many students could formulate linear equations in one unknown from simple contexts and demonstrate understanding of the meaning of roots of equations.

- Linear Equations in Two Unknowns: Students did well in formulating simultaneous equations from simple contexts and demonstrating recognition that graphs of equations of the form $ax + by + c = 0$ are straight lines. Students could solve a system of simple linear simultaneous equations by the graphical method or by algebraic methods. However, a small proportion of students could not do correct computation or provide complete solution. Their performance was only fair in plotting graphs of linear equations in 2 unknowns according to the values in the table.

Q44/M1
Example of Student Work (Did not draw a straight line to pass through the three points) 
Example of Student Work (Did not recognize that the graph of $ax + by + c = 0$ is a straight line) 

Q42/M1

Example of Student Work (Solving simultaneous equations – although the student knew how to use the method of substitution, mistakes occurred in the computation)

$2x+y=11$ ... (1) $x=2y+3$ ... (2)

Put (2) into (1)

$2(2y+3)+y=11$

$4y+3+y=11$

$4y+y=11-3$

$\frac{5y}{5}=\frac{8}{5}$

$y=1.6$ ... (3)

Put (3) into (2)

$x=2(1.6)+3$

$x=3.2+3$

$x=6.2$

$\therefore y=1.6$, $x=6.2$ //

Example of Student Work (Solving simultaneous equations – only *y* was solved)

$2x+y=11$ —— (1)

$x=2y+3$ —— (2)

把(2)代入(1)

$2(2y+3)+y=11$

$4y+6+y=11$

$5y=5$

$y=1$ //

Example of Student Work (Correct solution)

$\begin{cases} 2x+y=11 \text{ —— ①} \\ x=2y+3 \text{ —— ②} \end{cases}$

We put ② into ①

$2(2y+3)+y=11$

$4y+6+y=11$

$5y=5$

$y=1$

Put $y=1$ into ②

$x=2(1)+3$

$x=5$

$\therefore y=1$ and $x=5$

- Identities: Almost half of the students were not able to distinguish equations from identities. There was room for improvement in using the difference of two squares to expand simple algebraic expressions.

- Formulas: The majority of students were able to find the value of a specified variable in the formula. However, there was room for improvement in manipulation and simplification of algebraic fractions and performing change of subject in simple formulas.

Q28/M2
Exemplar Item (Manipulate algebraic fractions) Simplify $\frac{1}{4f}-\frac{1}{6f}$.
Example of Student Work (Wrong computation) (1) $-\frac{1}{f}$ (2) $\frac{1}{2f}-\frac{1}{3f}$ (3) $\frac{1}{12}$ (4) $12f$ (5) $\frac{2}{24f}$ (6) $\frac{1}{2f}$

- Linear Inequalities in One Unknown: The performance of students was good. They were able to use inequality signs to compare numbers and demonstrate good recognition of the properties of inequalities. However, a small proportion of students found difficulty in presenting the solution of simple linear inequalities in one unknown. Their performance was fair in formulating linear inequalities in one unknown from simple contexts.

Secondary 3 Measures, Shape and Space Dimension

S.3 students performed steadily in this dimension. They were able to perform simple calculations regarding transformation and symmetry, angles related with lines and rectilinear figures, trigonometric ratios and quadrilaterals. However, more improvement could be shown in items related to introduction to geometry and deductive geometry. Comments on students' performances are provided with examples cited where appropriate (question number *x* /sub-paper *y* quoted as Q*x*/M*y*). More items may also be found in the section ***General Comments***.

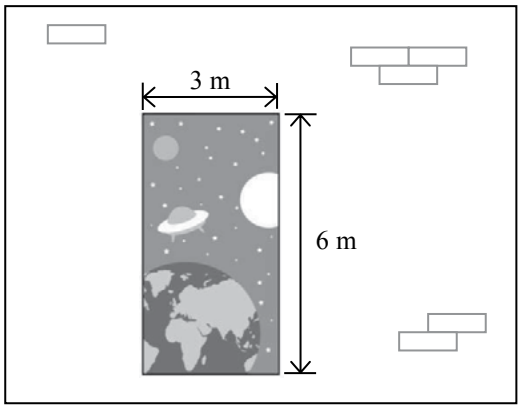
Measures in 2-D and 3-D Figures

- Estimation in Measurement: The majority of students were able to choose an appropriate unit and the degree of accuracy for real-life measurements and find the range of measures from a measurement of a given degree of accuracy. Their performance in selecting the appropriate ways to reduce errors in measurements was good. There was room for improvement in estimating measures with justification.

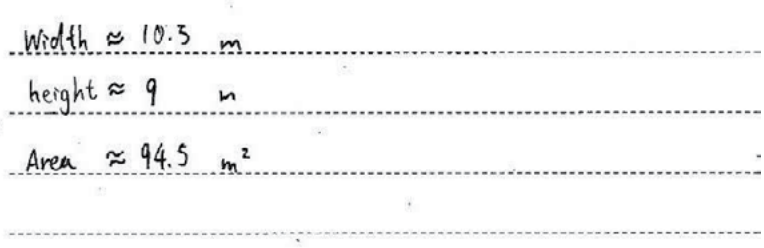
Q45/M3

Exemplar Item (Estimate the area of the wall)

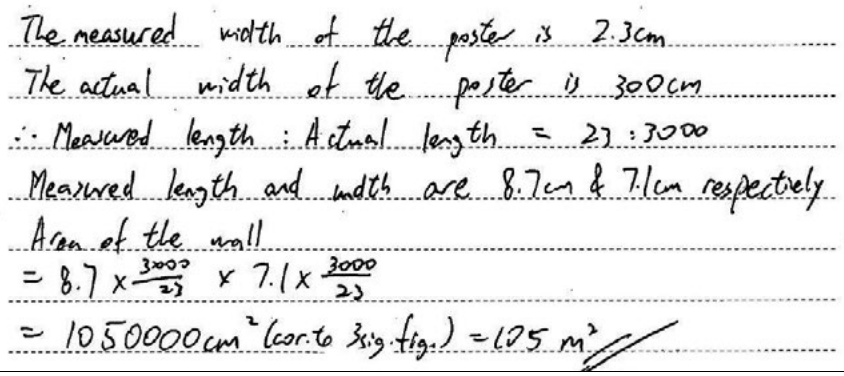
In the figure, a giant poster is hanging on a wall. The length and width of the poster are 6 m and 3 m respectively. Estimate the area of the wall and explain your estimation method.



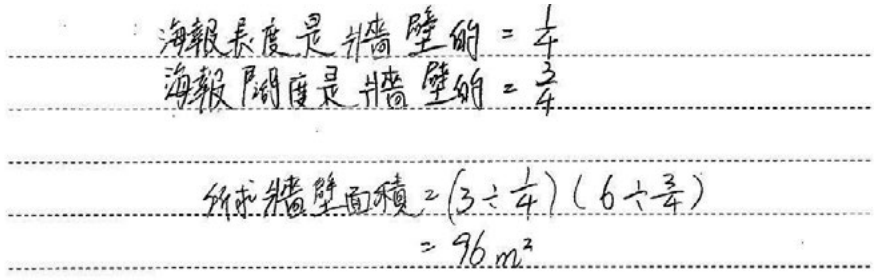
Example of Student Work (Without working steps or written explanation to show the estimation strategies in finding areas)



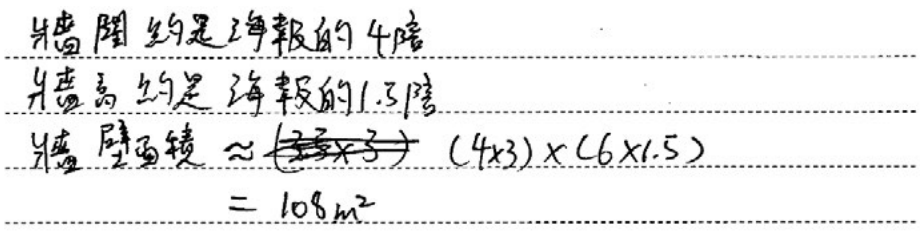
Example of Student Work (Calculated the area of the wall by measurement and conversion)



Example of Student Work (Estimation strategies did not match with the height and the width of the poster)



Example of Student Work (Estimated with reasonable justification)



- Simple Idea of Areas and Volumes: Quite a number of students were able to use the formulas for circumferences and curved surface areas of cylinders. Their performance in using the formula for volumes of prisms was fair.

- More about Areas and Volumes: Many students were capable of calculating arc lengths, areas of sectors, volumes of pyramids and curved surface areas of cones. However, only almost half of the students were able to use the relationships between sides and volumes of similar figures to solve related problems, and distinguish among formulas for volumes by considering dimensions.

Learning Geometry through an Intuitive Approach

- Introduction to Geometry: The majority of students were able to identify 3-D solids from given nets. Many of them could sketch simple solids. Quite a number of

students could use common notations to represent line segments. Half of the students were able to identify types of angles with respect to their sizes and identify cross-sections of given solids. However, they were weak in identifying concave polygons.

- Transformation and Symmetry: Many students were capable of determining the order of rotational symmetry from a figure. Quite a number of them could also identify the image of a figure and the effect on the size and shape under a single transformation. Nevertheless, their performance in determining the number of axes of symmetry was fair.

- Congruence and Similarity: A considerable number of students demonstrated recognition of the conditions for congruent triangles. They could apply the properties of congruent and similar triangles to find the sizes of angles and the lengths of sides in general. However, half of the students confused the reasons for similar triangles with that for congruent triangles when identifying two similar triangles.

- Angles related with Lines and Rectilinear Figures: Students demonstrated good recognition of interior angles of polygons and corresponding angles. They did well in solving simple geometric problems like using the angle properties associated with intersecting lines/parallel lines, using the relations between sides and angles associated with isosceles triangles and using the formula for the sums of the interior angles of convex polygons. Nevertheless, their performance in using the properties of angles of triangles to solve problems was fair.

- More about 3-D Figures: Students were able to identify axes of rotational symmetries of cubes, demonstrate recognition of planes of reflectional symmetries of cubes, identify nets of a right prism with equilateral triangles as bases and match 3-D objects built up of cubes from 2-D representations from various views. More than half of the students were able to name the projection of an edge on a horizontal plane and the angle between 2 planes.

Learning Geometry through a Deductive Approach

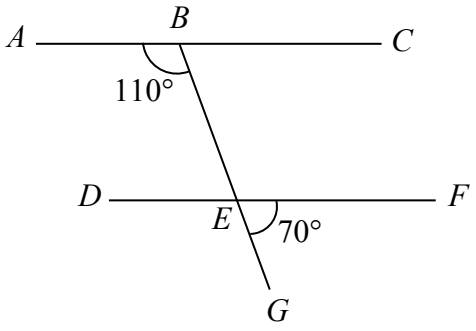
- Simple Introduction to Deductive Geometry: More than half of the students were able to identify perpendicular bisectors of a triangle. They could write the correct steps of a geometric proof and use the conditions for congruent triangles to perform simple proofs, but many of them still could not provide sufficient reasons or

complete the proof correctly.

Q46/M3

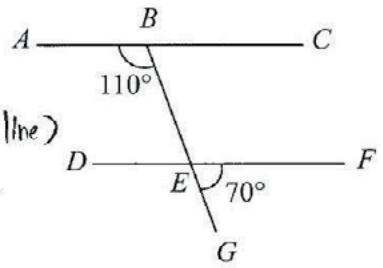
Exemplar Item (Geometric proof)

In the figure, ABC, DEF and BEG are straight lines. $\angle ABE = 110°$ and $\angle FEG = 70°$. Prove that $AC // DF$.



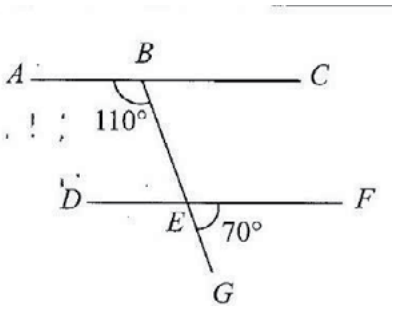
Example of Student Work (Incorrect logical reasoning in the proof – used the conclusion $AC // DF$ as a reasoning)

If FD// AC
∠CBE = ∠FEG (corr ∠s FD//AC)
180 = ABE + ∠CBE (∠s on a st. line)
∵ 110 + 70 = 180
∴ AC // DF



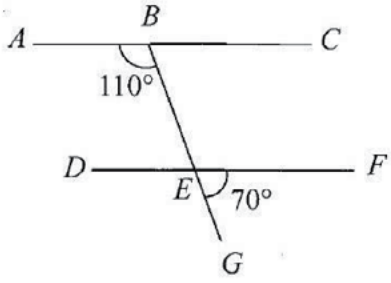
Example of Student Work (Not able to provide sufficient reasons)

∠BED = 70° (對頂角)
∠ABE + ∠BED = 180°
∴ AC // DF (同旁內角)



Example of Student Work (Good performance)

70° + ∠FEB = 180° (adj ∠s on st. line)
∠FEB = 110°
∵ ∠ABE = ∠FEB = 110°
∴ AC // DF (alt ∠s, equal) //

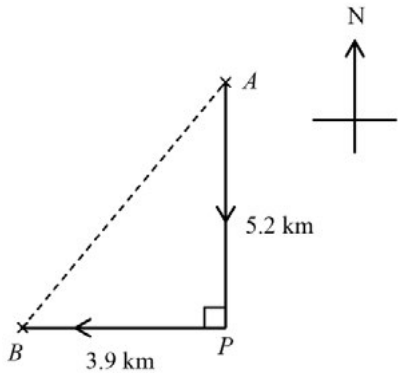


- Pythagoras' Theorem: Many students were able to use Pythagoras' Theorem to solve simple problems, but the performance for some students was only fair in determining whether the given triangles were right-angled triangles or not by using the converse of Pythagoras' Theorem.

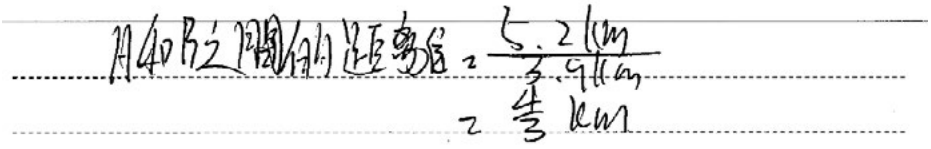
Q41/M4

Exemplar Item (Pythagoras' Theorem)

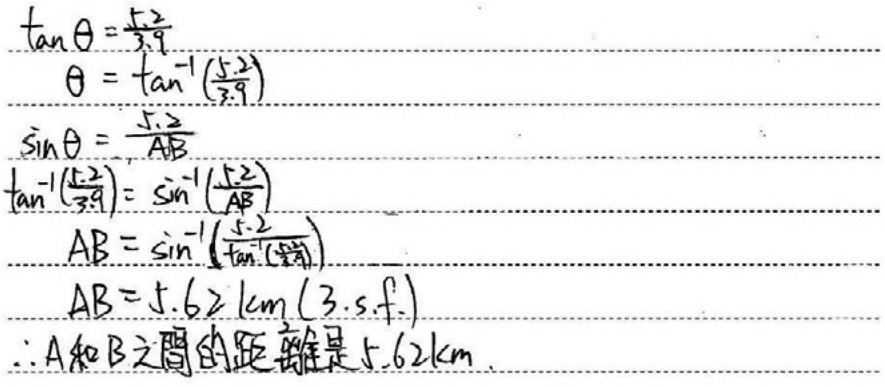
Tina walks due south for 5.2 km from A to P. Then, she walks due west for 3.9 km to B. Find the distance between A and B.



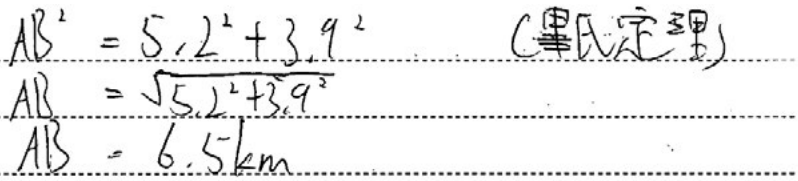
Example of Student Work (Confused the slope with the distance between A and B)



Example of Student Work (Wrong computation)



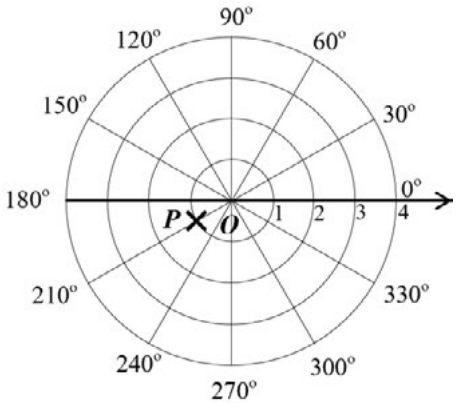
Example of Student Work (Good performance)



- Quadrilaterals: The performance of students in using the properties of rectangles in numerical calculations was good.

Learning Geometry through an Analytic Approach

- Introduction to Coordinates: Students were good at grasping the basic concepts of the rectangular coordinate system, but their recognition of polar coordinates was fair. They were able to match a point under a single transformation with its image in the rectangular coordinate plane. Their performance was fair in calculating areas of simple figures although there was a slight improvement.

Q35/M4
Exemplar Item (Find the polar coordinates) Find the polar coordinates of point ***P*** in the figure. 
Example of Student Work (1) ***P*** 點的極坐標是 (1 , 210°)。 (Missing the unit) (2) The polar coordinates of point ***P*** are (210° , 1). (Not able to write the correct ordered pair)

- Coordinate Geometry of Straight Lines: Many students were able to use distance formula and the formula of finding slopes. However, their performance was only fair in using the mid-point formula and demonstrating recognition of the conditions for perpendicular lines.

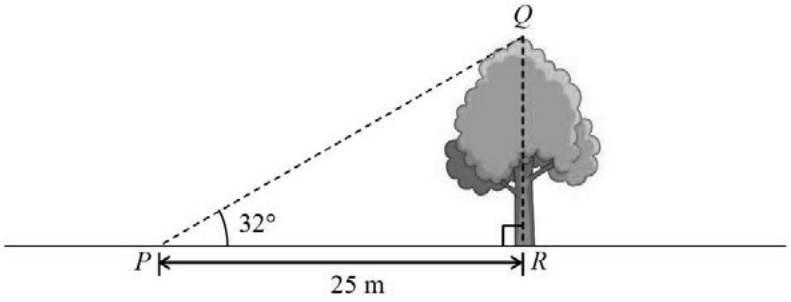
Trigonometry

- Trigonometric Ratios and Using Trigonometry: Many students were able to grasp the basic concepts of trigonometric ratios, solve simple 2-D problems involving one right-angled triangle and demonstrate recognition of the angle of elevation.

Q42/M3

Exemplar Item (Find the height of the tree)

In the figure, Thomas is standing at point P. The horizontal distance between Thomas and a tree QR is 25 m. The angle of elevation of the top Q of the tree from point P is 32°. Find the height of the tree QR. (Correct to 3 significant figures)



Example of Student Work (Confused opposite sides with adjacent sides)

$\tan 32° = \frac{25}{QR}$

$QR \tan 32° = 25$

$QR = \frac{25}{\tan 32°}$

$QR \approx 40.0$

The required ans is 40.0m

Example of Student Work (Confused tangent ratios with sine ratios)

let QR be the height of the tree

$\sin 32 = \frac{QR}{25}$

$25 \sin 32 = QR$

$QR \approx 13.2$

∴ The height of the tree is 13.2 cm

Example of Student Work (Good performance)

$\tan 32° = \frac{QR}{25}$

$QR = 25 \tan 32°$

$QR = 15.6\text{m}$ ✓

Secondary 3 Data Handling Dimension

The performances of S.3 students were quite good in this dimension. They were able to interpret statistical charts, organize the same set of data by different grouping methods, find mean and median from a set of ungrouped data and calculate probabilities. However, performance was relatively weak in distinguishing discrete and continuous data. They were not able to identify sources of deception in cases of misuse of averages. Comments on students' performance are provided below with examples cited where appropriate (question number *x* / sub-paper *y* quoted as Q*x*/M*y*). More examples may also be found in the section ***General Comments***.

Organization and Representation of Data

- Introduction to Various Stages of Statistics: Students were able to demonstrate recognition of various stages of statistics and organize the same set of data by using different grouping methods. More than half of the students could use simple methods to collect data. However, quite a number of students could not distinguish between discrete and continuous data.

- Construction and Interpretation of Simple Diagrams and Graphs: Many students were capable of identifying sources of deception in misleading graphs/accompanying statements and interpreting scatter diagrams or broken line graphs. The majority of students could even choose appropriate diagrams/graphs to present a set of data and read median from diagrams/graphs. However, some students could not represent a time in the '24-hour time' when interpreting statistical charts. A small proportion of students could not construct simple statistical charts.

Q47/M2

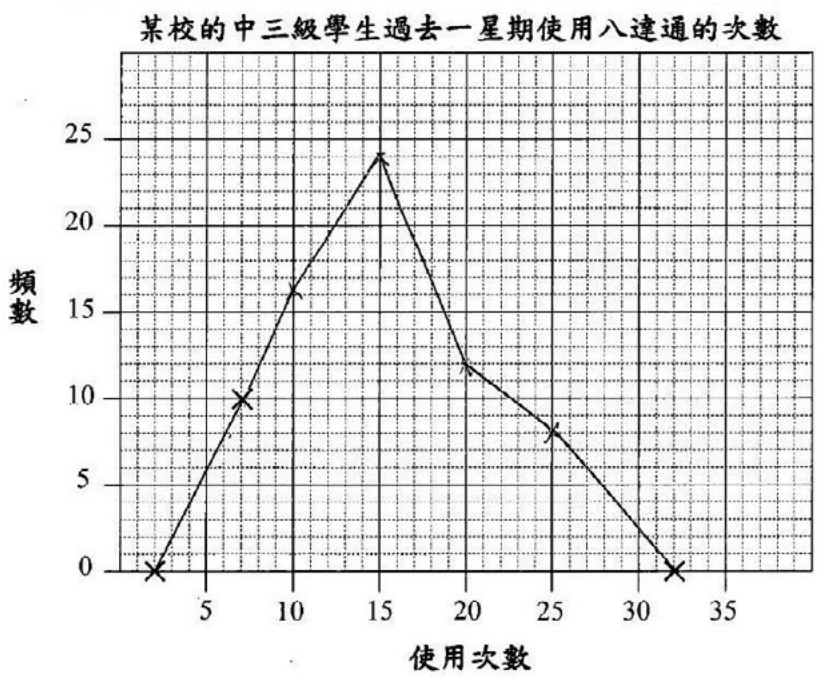
Exemplar Item (Draw a frequency polygon)

The table below shows the number of times Secondary 3 students of a school used their Octopus cards last week.

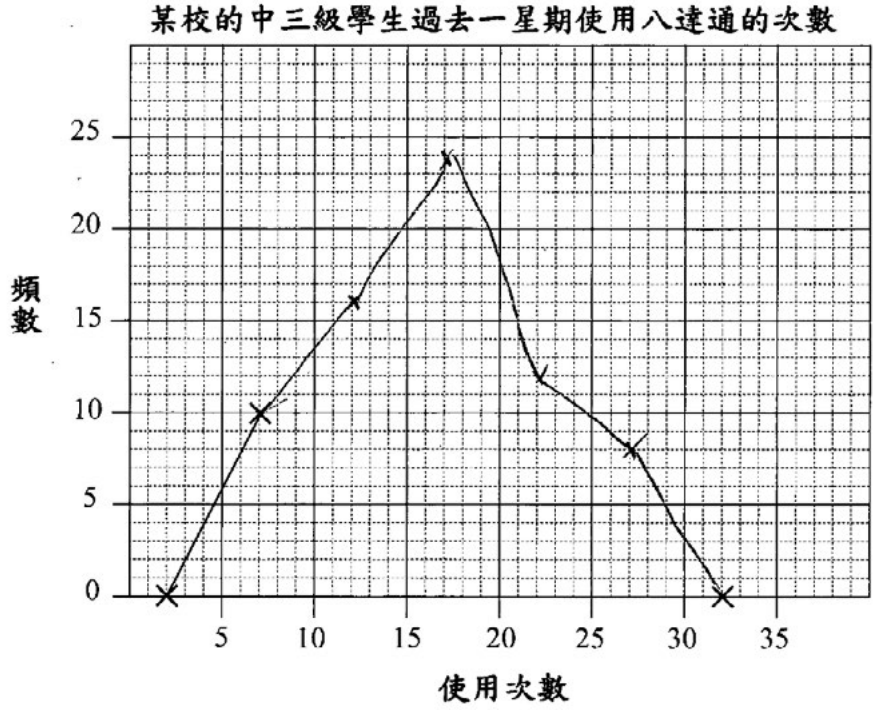
Number of times using Octopus card	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29
Class mark	7	12	17	22	27
Frequency	10	16	24	12	8

Draw a frequency polygon in the **ANSWER BOOKLET** to represent the above data.

Example of Student Work (Did not plot the graph with correct class marks)



Example of Student Work (Did not join all the points with straight lines)



Analysis and Interpretation of data

- Measures of Central Tendency: The majority of students were able to find the mean and median from a set of ungrouped data. Many students could find modal class from a set of grouped data and calculate the weighted mean of a set of data. However, they were quite weak in identifying sources of deception in cases of misuse of averages.

Q47/M3

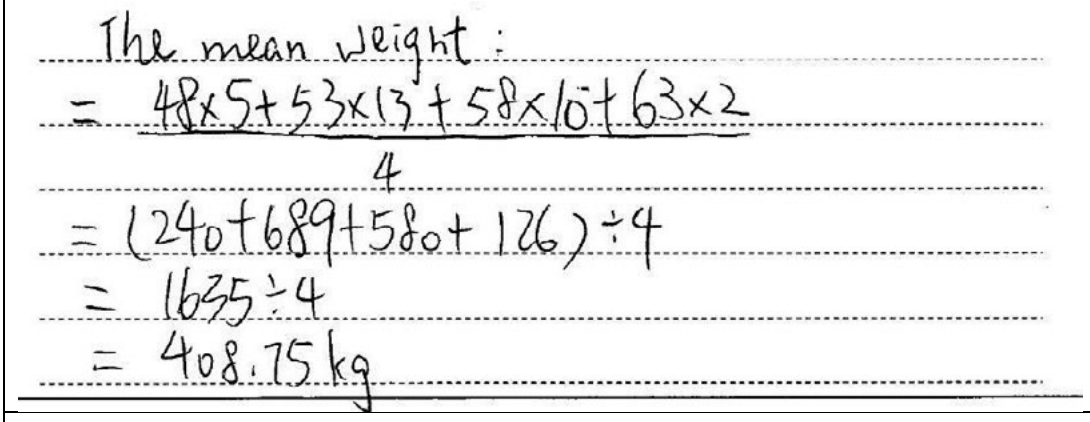
Exemplar Item (Find mean from a set of grouped data)

The table below shows the weights of 30 customers of a fitness centre.

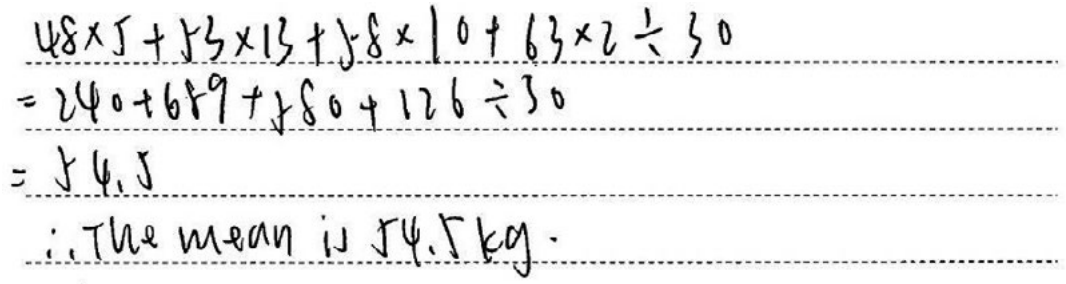
Weight (kg)	46 – 50	51 – 55	56 – 60	61 – 65
Frequency	5	13	10	2

Find the mean weight of the 30 customers.

Example of Student Work (Did not consider the frequency after calculating the total weight of the customers)



Example of Student Work (Missing the parentheses)



Q47/M1

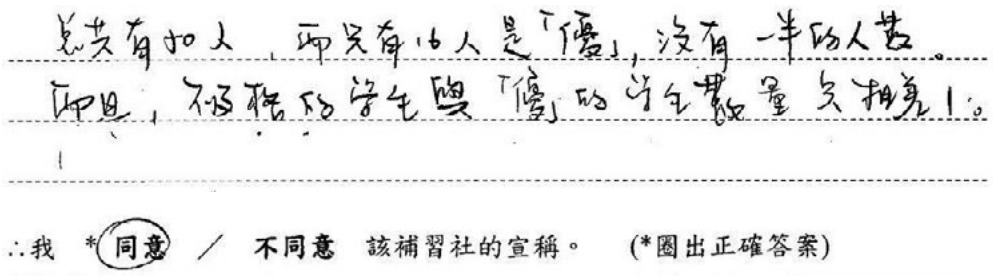
Exemplar Item (Identify sources of deception)

The following table shows the mathematics results of 50 students of a private tuition institute in a public examination.

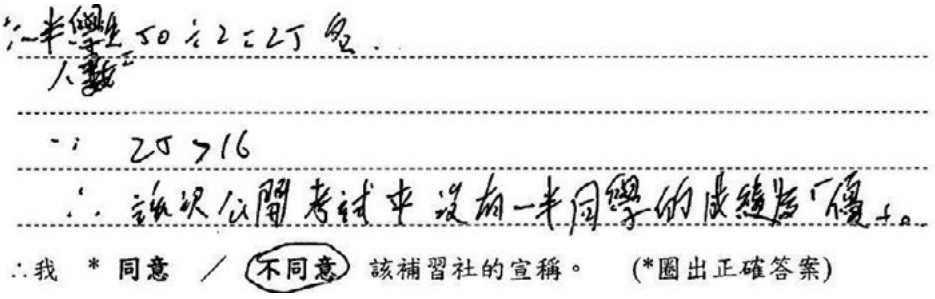
Grade	Distinction	Credit	Pass	Fail
Number of students	16	8	11	15

It is given that the mode of the results of these 50 students is "Distinction". Hence a claim "more than half of the students got distinction in that Mathematics public examination" is made by the tuition institute.
Do you agree on the claim made by the tuition institute? Explain your answer.

Example of Student Work (Stating the information given only without further explanation as to why the student agreed with the tuition institute's claim)



Example of Student Work (Good performance)



Probability

- Simple Idea of Probability: The performance of students was quite good in calculating the empirical probability and theoretical probability, in particular with calculating the theoretical probability by listing.

General Comments on Secondary 3 Student Performances

The overall performance of S.3 students was steady. They did quite well in the Data Handling Dimension. Performance was steady in the Number and Algebra Dimension and Measures, Shape and Space Dimension.

The areas in which students demonstrated adequate skills are listed below:

Directed Numbers and the Number Line

- Demonstrate recognition of the ordering of integers on the number line (e.g. Q21/M1).
- Add, subtract, multiply and divide directed numbers (e.g. Q21/M4).

Numerical Estimation

- Judge, without actual calculations, the reasonableness of answers from computations (e.g. Q1/M1).

Approximation and Errors

- Convert numbers in scientific notation to integers or decimals (e.g. Q2/M2).

Using Percentages

- Solve simple selling problems (e.g. Q40/M2).

Rate and Ratio

- Use rate and ratio to solve simple real-life problems (e.g. Q43/M3).

Formulating Problems with Algebraic Language

- Formulate simple equations/inequalities from simple contexts (e.g. Q2/M4).
- Describe patterns by writing the next few terms in arithmetic sequences, geometric sequences, Fibonacci sequence or sequences of polygonal numbers from several consecutive terms of integral values (e.g. Q24/M3).

Manipulations of Simple Polynomials

- Multiply a trinomial by a monomial (e.g. Q25/M4).

Laws of Integral Indices

- Find the value of a^n, where a and n are integers (e.g. Q7/M4).

Linear Equations in One Unknown

- Solve simple equations (e.g. Q28/M1).

Linear Equations in Two Unknowns

- Formulate simultaneous equations from simple contexts (e.g. Q6/M1).

Linear Inequalities in One Unknown

- Use inequality signs $\geq$, $>$, $\leq$ and $<$ to compare numbers (e.g. Q30/M2).
- Represent inequalities, such as $x < -2$, $x \geq 3$, etc., on the number line and vice versa (e.g. Q8/M3).

Estimation in Measurement

- Find the range of measures from a measurement of a given degree of accuracy (e.g. Q9/M2).
- Reduce errors in measurements (e.g. Q9/M1).

Introduction to Geometry

- Make 3-D solids from given nets (e.g. Q13/M1).

Transformation and Symmetry

- Name the single transformation involved in comparing the object and its image (e.g. Q14/M1).

Congruence and Similarity

- Demonstrate recognition of the properties of congruent and similar triangles (e.g. Q33/M2).

Angles related with Lines and Rectilinear Figures

- Demonstrate recognition of terminologies on angles with respect to their positions relative to lines and polygons (e.g. Q15/M4).
- Use the angle properties associated with intersecting lines/parallel lines to solve simple geometric problems (e.g. Q33/M4).

- Use the relations between sides and angles associated with isosceles/equilateral triangles to solve simple geometric problems (e.g. Q34/M2).
- Use the formulas for the sums of the interior angles and exterior angles of convex polygons (e.g. Q34/M1).

More about 3-D Figures

- Match 3-D objects built up of cubes from 2-D representations from various views (e.g. Q16/M1).

Quadrilaterals

- Use the properties of parallelograms, squares, rectangles, rhombuses, kites and trapeziums in numerical calculations (e.g. Q33/M3).

Introduction to Coordinates

- Use an ordered pair to describe the position of a point in the rectangular coordinate plane and locate a point of given rectangular coordinates (e.g. Q34/M3).

Introduction to Various Stages of Statistics

- Organize the same set of data by different grouping methods (e.g. Q37/M2).

Construction and Interpretation of Simple Diagrams and Graphs

- Interpret simple statistical charts (e.g. Q38/M1 and Q38/M3).
- Compare the presentations of the same set of data by using statistical charts (e.g. Q 19/M1).

Measures of Central Tendency

- Find mean, median and mode from a set of ungrouped data (e.g. Q39/M1).
- Find median, mean and modal class from a set of grouped data (e.g. Q47/M3).

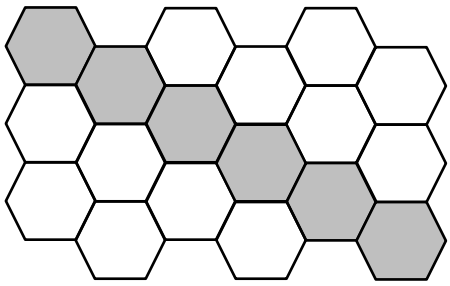
Simple Idea of Probability

- Calculate the theoretical probability by listing (e.g. Q47/M4).

Other than items in which students performed well, the assessment data also provided some entry points to strengthen learning and teaching. Items worthy of attention are discussed on the following pages:

Rate and Ratio

- Represent a ratio in the form $a : b$ (or $\frac{a}{b}$), $a : b : c$ (e.g. Q2/M1): Almost half of the students chose the correct answer, option A. However, more than 40% of students chose option B. Those students may not have read the question carefully and mistakenly found the ratio of the number of shaded hexagons to the total number of hexagons.

Q2/M1
 In the figure, there are 18 identical hexagons and 6 of them are shaded. Find the ratio of the number of shaded hexagons to the number of hexagons in white. A. 1 : 2 B. 1 : 3 C. 2 : 1 D. 2 : 3

Formulating Problems with Algebraic Language

- Distinguish the difference between $2x$ and $2 + x$; $(-2)^n$ and -2^n; x^2 and $2x$, etc (e.g. Q3/M1): Half of the students chose the correct answer, option A, but more than 30% of students chose options C. They mistakenly took $(-y)^2 = -y^2$.

Q3/M1
$-y^2 + (-y)^2 =$ A. 0. B. $2y^2$. C. $-2y^2$. D. y^4.

Manipulations of Simple Polynomials

- Demonstrate recognition of terminologies (e.g. Q4/M3): More than half of the students could not choose the correct answer, option A. Each of the remaining options was chosen by more than 10% of students. For those who chose options B, C or D, they were not able to demonstrate recognition of the degrees of polynomials. They confused degrees with leading coefficients, constants or number of terms of polynomials respectively.

Q4/M3
Find the degree of the polynomial $6y^7 + y^2 - y + 5$. A. 7 B. 6 C. 5 D. 4

Identities

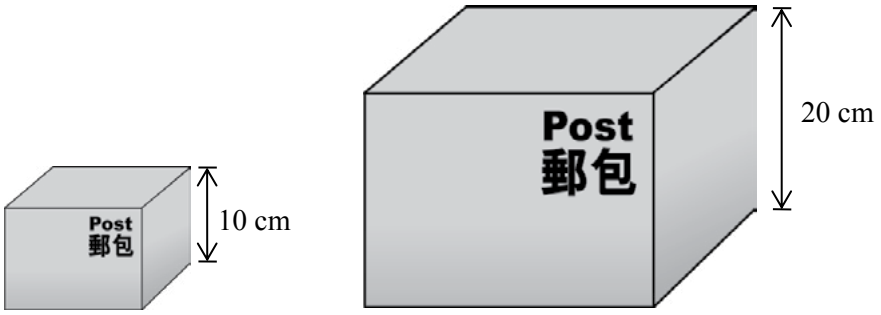
- Tell whether an equality is an equation or an identity (e.g. Q7/M1): More than half of the students chose the correct answer, option C. However, option B was chosen by about 20% of students. Those students were not able to distinguish the difference between equations and identities. In addition, options A and D were chosen by about 10% of students respectively. They mistakenly took $3(x + a) = 3x + a$ or $x^2 + a = (x + a)(x - a)$ as identities, where $a \neq 0$.

Q7/M1
Which of the following is an identity? A. $3(x + 4) = 3x + 4$ B. $\frac{3x-1}{2} = 4$ C. $x - 3 = -(3 - x)$ D. $x^2 + 3 = (x + 3)(x - 3)$

More about Areas and Volumes

- Use the relationships between sides and surface areas/volumes of similar figures to solve related problems (e.g. Q11/M2): Almost half of the students chose the correct answer, option D, but more than 30% of students still chose option A. They mistakenly took the ratio of the volumes of similar figures as the ratio of their corresponding heights. Moreover, more than 10% of students chose B. They mistakenly thought that the ratio of the volumes of Parcel *A* and Parcel *B* as the ratio of the squares of their corresponding heights.

Q11/M2
In the figure, Parcel A and Parcel B are similar solids. Their heights are 10 cm and 20 cm respectively. The volume of Parcel A is 1 200 cm^3. Find the volume of Parcel B.

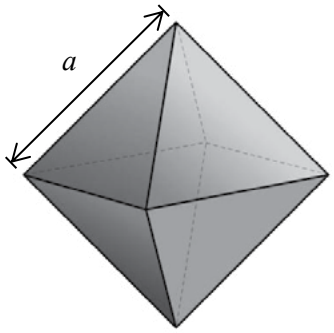


Parcel *A* Parcel *B*

A. 2 400 cm^3
B. 4 800 cm^3
C. 7 200 cm^3
D. 9 600 cm^3

- Distinguish among formulas for lengths, areas, volumes by considering dimensions (e.g. Q9/M4): Nearly half of the students chose the correct answer, option A. More than half of the students chose the distractors. They were not able to identify the given formula for the volume of the octahedron by considering dimensions.

Q9/M4



The solid in the figure is an octahedron. Each of its side length is a. By considering the **dimensions**, determine which of the following could express the volume of the above figure.

A. $\frac{\sqrt{2}a^3}{3}$

B. $2\sqrt{3}a^2$

C. $\frac{a\sqrt{10a^2+7a}}{2}$

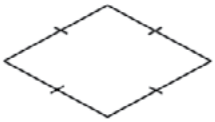
D. $\frac{\sqrt{2}a}{2}$

Introduction to Geometry

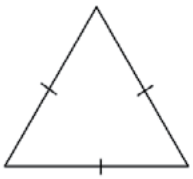
- Demonstrate recognition of common terms in geometry (e.g. Q11/M1): Less than 30% of students chose the correct answer, option C. However, each of the remaining options was chosen by more than 20% of students. Those students mistakenly identified the rhombus, the rectangle or the figure in option D as a regular polygon.

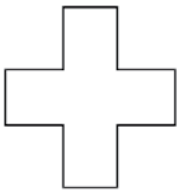
Q11/M1

Which of the following figures can represent a regular polygon?

A. 

B. 

C. 

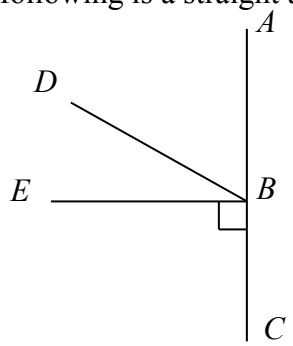
D. 

- Identify types of angles with respect to their sizes (e.g. Q10/M3): Half of students chose the correct answer, option C. However, option B was chosen by more than

30% of students. Those students mistakenly identified the right angle $\angle CBE$ as a straight angle.

Q10/M3

In the figure, ABC is a straight line. Which of the following is a straight angle?



A. $\angle CBD$

B. $\angle CBE$

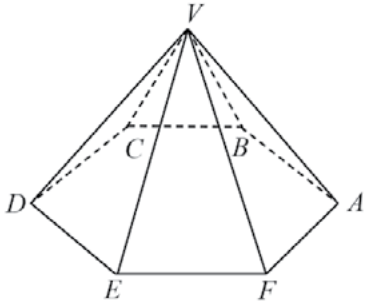
C. $\angle ABC$

D. $\angle ABD$

- Sketch cross-sections of simple solids (e.g. Q11/M4): More than half of the students chose the correct answer C. More than 20% of students chose option B and more than 10% of students chose option D. They were not able to identify the corresponding plane diagram of the cross-section according to the given right pyramid.

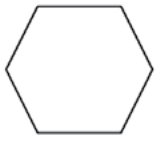
Q11/M4

A right pyramid is placed horizontally as shown. Its base $ABCDEF$ is a regular hexagon. Max sketches a cross-section which is perpendicular to the base and passing through vertex V.



Which of the following could express the plane diagram of the cross-section?

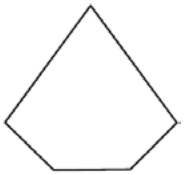
A.



B.

C.

D.



Coordinate Geometry of Straight Lines

- Demonstrate recognition of the conditions for parallel lines and perpendicular lines (e.g. Q18/M4): More than half of students chose the correct answer, option C. However, option B was chosen by more than 10% of students. Those students confused the conditions for perpendicular lines with those for parallel lines.

Q18/M4

It is given that the slope of a straight line ℓ is $\frac{5}{9}$. Which of the following straight lines is perpendicular to ℓ?

Line	L_1	L_2	L_3	L_4
Slope	$-\frac{5}{9}$	$\frac{5}{9}$	$-\frac{9}{5}$	$\frac{9}{5}$

A. L_1
B. L_2
C. L_3
D. L_4

Good Performance of Secondary 3 Students in 2019

- Students with good performance demonstrated mastery of the concepts and skills assessed by the sub-papers. Their performance in numeracy skills and problem-solving skills was good, and they were able to solve various types of problems relating to directed numbers, numerical estimation, rate and ratio. Students had a thorough conceptual understanding of algebra and could observe patterns and express generality. They were able to deal with the basic operations, factorization and expansion of simple polynomials, and were familiar with linear inequalities in one unknown. They were capable of solving equations by using algebraic and graphical methods. They could also plot graphs of linear equations in two unknowns.

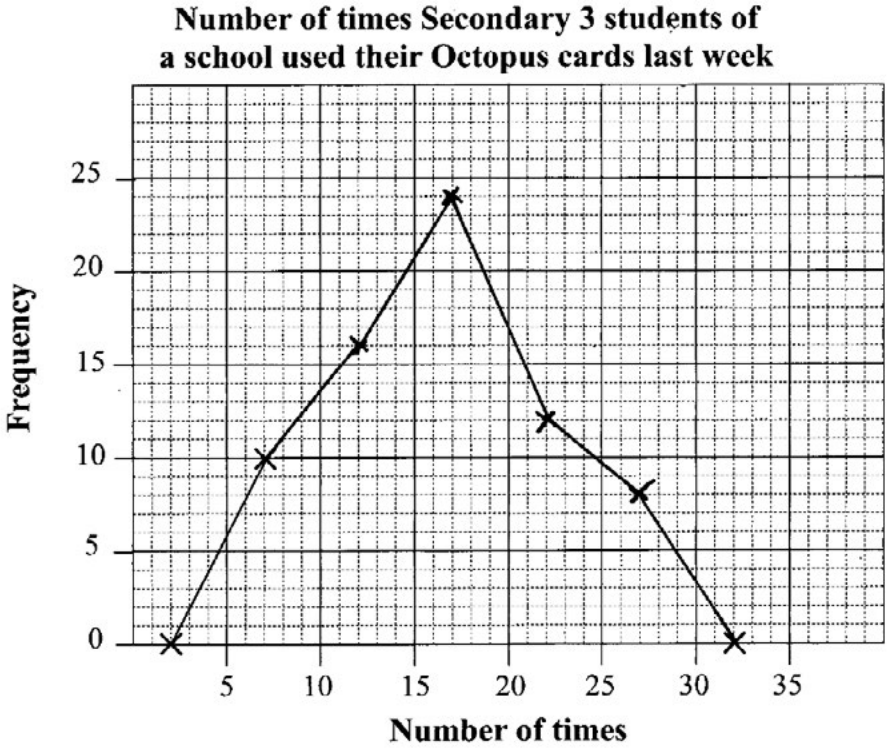
- Students with good performance were also capable of calculating the areas of simple plane figures and the surface areas and volumes of some solids. They were able to demonstrate good recognition of the concepts of transformation and symmetry, congruence and similarity, coordinate geometry, quadrilaterals, trigonometry, and Pythagoras' Theorem. They were able to complete the geometric proofs with the correct steps and sufficient reasons provided.

- Students with good performance had a good knowledge of the various stages of statistics and grasp the basic concepts of probability. They were able to construct and interpret simple statistical charts, use statistical charts appropriately, read information from graphs, find the mean, median and mode/modal class, as well as identify sources of deception from a set of data.

- The examples of work by these students are illustrated as follows:

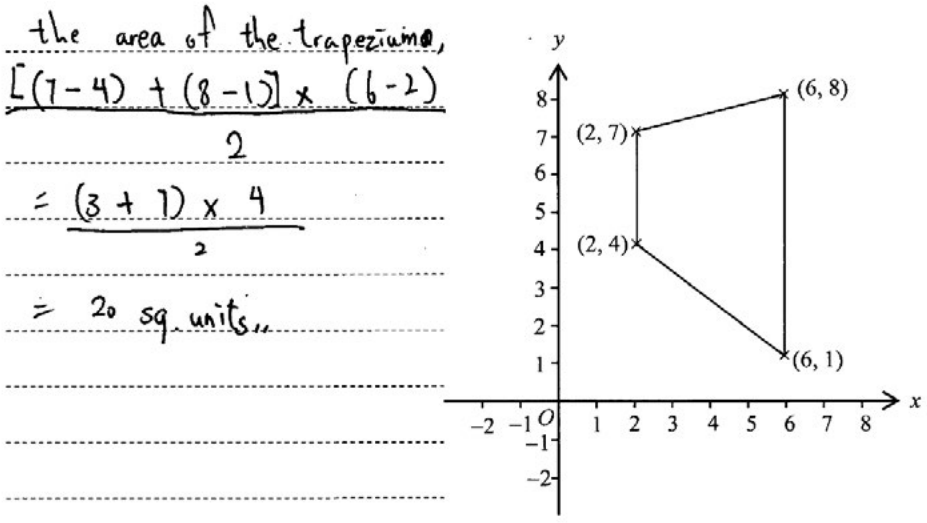
Students were able to construct simple statistical charts by using the given data.

Q47/M2
Example of Student Work (Construct simple statistical charts)

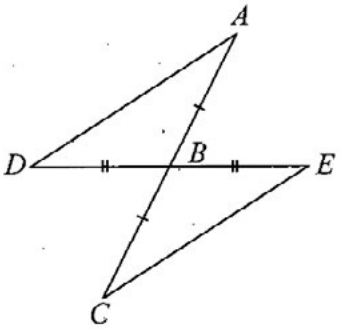


Students were able to solve the problem correctly with complete and clear presentation.

Q42/M4
Example of Student Work (Find the area of a trapezium)



Students were able to make good use of the given conditions and solve the problem systematically.

Q46/M1
Example of Student Work (Geometric proof) ∠ABD = ∠CBE (vert. opp. ∠s) AB = CB (given) BD = BE (given) ∴ △ABD ≅ △CBE (SAS) 

Some common weaknesses of high-achieving students were that:

- Some students were not able to determine whether a polygon is regular.
- Some students were not able to determine whether a polygon is concave.
- Some students were not able to distinguish discrete and continuous data.

Overview of Secondary 3 Student Performances in Mathematics in 2017-2019

The percentage of students achieving Basic Competency in the Territory-wide System Assessment this year was 79.6%.

The percentages of students achieving Basic Competency from 2017 to 2019 are listed below:

Table 8.9 Percentages of S.3 Students Achieving Mathematics Basic Competency in 2017 - 2019

Year	% of Students Achieving Mathematics Basic Competency
2017	79.9
2018	80.0
2019	79.6

The performances of S.3 students over the past three years in each dimension of Mathematics are summarized in the following table:

Table 8.10 Overview of S.3 Student Performances in Mathematics in 2017-2019

Year / Number and Algebra	2017	2018	2019	Remarks
Strengths	• Students did well in the operations of directed numbers. They demonstrated recognition of the number line. They could also use directed numbers to describe real-life situations. • Students were able to solve simple problems on depreciations. • Students were able to convert numbers in scientific notation to integers and round off a number to 3 significant figures. • Students were able to solve simple problems by using rate. • Students were able to solve a system of linear simultaneous equations by algebraic methods. • Students were able to substitute values into formulas to find the unknown values. • Students demonstrated recognition of inequalities.	• Students were good at using directed numbers to describe real life situations. They also recognized the ordering of integers on the number line. • Students could determine whether to estimate or to compute the exact value in a simple context. • Students did well in representing real numbers on the number line. • Students were able to solve simple problems by using ratio. • Students were able to substitute values into formulas to find the unknown values. • Students demonstrated good recognition of inequalities.	• Students did well in the operations of directed numbers. They demonstrated recognition of the ordering of integers on the number line. • Students were capable of converting numbers in scientific notation to integers. • Students were good at formulating simple inequalities from simple contexts. • Students are able to describe patterns by writing the next few terms in arithmetic sequences from several consecutive terms of integral values. • Students could solve simple equations. • Students demonstrated good recognition of using inequality signs to compare numbers.	• Students always used rounding off for estimation without considering the actual requirements of questions. • A small proportion of students were weak in some basic mathematical concepts like the operations of fractions, use of parentheses etc. • Units were often omitted in the answer. • Students were willing to show their working steps and strategies used in solving problems, but sometimes the solutions were incomplete.

Year / Number and Algebra	2017	2018	2019	Remarks
Weaknesses	• Quite a number of students were not able to estimate values with reasonable justifications. • Students mixed up the formulas for finding simple interest and compound interest. • Quite a number of students were not able to distinguish polynomials from algebraic expressions. • Students were weak in recognizing the terminologies of polynomials. • Students' performance was only fair in change of subject in simple formulas.	• Quite a number of students were not able to estimate values according to the given context with reasonable justifications. • Quite a number of students were not able to distinguish the difference between $(-2)^n$ and -2^n. • Students were weak in recognizing the terminologies of polynomials such as number of terms. • Students were quite weak in recognizing the meaning of roots of equations. • Students' performance was not satisfactory in manipulating algebraic fractions.	• Many students were not able to estimate values according to the given context with reasonable justifications. • Half of the students were not able to solve problems on simple interest to find the interest rate. • Students' performance was not satisfactory in expanding simple algebraic expressions by using the perfect square expressions. • Students were weak in performing change of subject in simple formulas. • Students' performance was only fair in using the laws of integral indices to simplify simple algebraic expressions. • Students' recognition of plotting graphs of linear equations in 2 unknowns was insufficient.	

Year / Measures, Shape and Space	2017	2018	2019	Remarks
Strengths	• Students were able to find the range of measures from a measurement of a given degree of accuracy and estimate measures with justification. • Students were able to select the appropriate ways to reduce errors in measurements. • Students were able to use the formulas of volumes of prisms, find the areas of sectors and the total surface areas of pyramids. • Students were able to identify the relationship between simple 3-D solids and their corresponding 2-D figures. • Students were able to demonstrate recognition of the concepts of transformation and symmetry. • Students were able to use the angle properties associated with intersecting lines/parallel lines and the properties of triangles to solve simple geometric problems. • Students were familiar with the properties of parallelograms. • Students had good knowledge of the rectangular coordinate system.	• Students were able to choose an appropriate unit and the degree of accuracy for real-life measurements. • Students were able to find the volumes of cylinders. • Students could use notations to represent angles. • Students demonstrated good recognition of the concepts of transformation. • Students were able to identify the nets of cubes. • Students were good at using the properties of squares in numerical calculations. • Students had good knowledge of the rectangular coordinate system. • Students understood the basic concepts of trigonometric ratios.	• Students were able to choose an appropriate unit and the degree of accuracy for real-life measurements. • Students were able to select the appropriate ways to reduce errors in measurements. • Students were able to identify 3-D solids from given nets. • Students were capable of demonstrating recognition of interior angles of polygons. • Students were able to use the angle properties associated with intersecting lines/parallel lines to solve simple geometric problems. • Students were able to use the formula for the sums of the interior angles of convex polygons. • Students did well in using the relations between sides and angles associated with isosceles triangles. • Students were good at using the properties of rectangles in numerical calculations. • Students had good knowledge of the rectangular coordinate system.	• In doing geometric proofs, some students used circular arguments and gave incorrect logical reasoning. Or, they could not use logical reasoning and correct reasons to complete the proofs. • Inappropriate or incorrect presentation frequently occurred such as confusing $\angle ABC$ with $\triangle ABC$, $AB = BC$ with $AB \parallel BC$. • Units were often omitted in the answer.

Year / Measures, Shape and Space	2017	2018	2019	Remarks
Weaknesses	• Students in general were unable to use relationship of similar figures to find measures and distinguish among formulas for areas by considering dimensions. • Many students were not able to determine whether a polygon is equilateral. • Students were quite weak in recognizing the conditions for congruent and similar triangles. • Students were weak in identifying the planes of reflectional symmetries of cubes. • Students in general were not able to complete the proofs of simple geometric problems. • Many students were not able to name the angle between a line and a plane. • Students' performance was only fair in applying the conditions for two perpendicular lines.	• Students' performance in using the relationships between sides and surface areas of similar figures to solve related problems was fair. • Many students were not able to determine whether a polygon is convex. • Students were not able to use the conditions for similar triangles to perform simple proofs. • Students in general were not able to complete the proofs of simple geometric problems.	• Students' performance in using the relationships between sides and volumes of similar figures to solve related problems was fair. • Students were unable to distinguish among formulas for volumes by considering dimensions. • Students were weak in identifying regular polygons and concave polygons. • Students were not able to identify whether two triangles are congruent/similar with simple reasons. • Students in general were not able to complete the proofs of simple geometric problems.	

Year / Data Handling	2017	2018	2019	Remarks
Strengths	• Students were able to use simple methods to collect data. • Students were able to interpret simple statistical charts. • Students were able to choose appropriate diagrams/graphs to present a set of data. • Students were able to find mean and median from a set of ungrouped data. • Students' performance was quite good in calculating probabilities.	• Students could organize the same set of data by different grouping methods. • Students could construct and interpret simple statistical charts. • Students were able to find mean and median from a set of ungrouped data. • Students could calculate the theoretical probability by listing.	• Students could organize the same set of data by different grouping methods. • Students could interpret simple statistical charts and compare the presentations of the same set of data by using statistical charts. • Students could calculate the theoretical probability by listing.	• Many students did not use rulers to draw statistical charts. • Students were willing to describe the sources of deception in cases of misuse of averages, but in general, they stated the given information only and were not able to give sufficient explanations.
Weaknesses	• Students' performance was only fair in distinguishing discrete and continuous data. • Students in general were not able to construct histograms correctly. • Quite a number of students were not able to identify sources of deception in cases of misuse of averages.	• Students could not choose appropriate diagrams/graphs to present a set of data in general. • The performance of students in identifying sources of deception in cases of misuse of averages was not satisfactory.	• Students were weak in distinguishing discrete and continuous data. • The performance of students in identifying sources of deception in cases of misuse of averages was not satisfactory.	

Comparison of Student Performances in Mathematics in Primary 3, Primary 6 and Secondary 3 in 2019

The percentages of P.3, P.6 and S.3 students achieving Basic Competency from 2006 to 2019 are as follows:

Table 8.11 Percentages of Students Achieving Mathematics Basic Competency

Year / Level	% of Students Achieving Mathematics BC													
	2006	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
P.3	86.9	86.9	86.9	#	87.0	87.0	87.3	87.5	87.4	87.6	89.9$^{\Delta}$	88.2$^{\nabla}$	88.0$^{\square}$	87.7$^{\square}$
P.6	83.8	83.8	84.1	#	84.2	84.1	^	84.2	^	84.0	^	84.0	^	84.2
S.3	78.4	79.9	79.8	80.0	80.1	80.1	79.8	79.7	79.9	79.9	80.0	79.9	80.0	79.6

\# Due to Human Swine Influenza causing the suspension of primary schools, the TSA was cancelled and no data was provided.

^ As participation in the P.6 TSA has been on a voluntary basis in even-numbered years since 2012, not all P.6 TSA were involved and hence no territory-wide data is provided in this report.

$^{\Delta}$ The 2016 P.3 level assessment was conducted as part of the 2016 Tryout Study. The BC attainment rate was calculated using the data from some 50 participating schools.

$^{\nabla}$ The 2017 P.3 level assessment was conducted as part of the 2017 Research Study, which was extended to all primary schools in the territory.

$^{\square}$ Starting from 2018, the P.3 TSA is conducted on a sampling basis. The BC attainment rates are inferred from the sample of all students participating in the assessment.

The data shows the performance of Primary 3, Primary 6 and Secondary 3 students in different dimensions. Teachers could adapt their teaching strategies and curriculum design in accordance with the strengths and weaknesses of students. The dimensions of Mathematics Curriculum at each key stage belong to different dimensions as shown below:

Table 8.12 Dimensions of Mathematics Curriculum for Primary 3, Primary 6 and Secondary 3

	Primary 3	**Primary 6**	**Secondary 3**
Dimension	Number	Number	Number and Algebra
		Algebra	
	Measures	Measures	Measures, Shape and Space
	Shape and Space	Shape and Space	
	Data Handling	Data Handling	Data Handling

The following table compares student performances in Mathematics in Primary 3, Primary 6 and Secondary 3 in 2019:

Table 8.13 Comparison of Student Performances in Mathematics in Primary 3, Primary 6 and Secondary 3 in 2019

Dimension \ Level	P.3	P.6	S.3
Number	• Students were able to recognize the places and the values of digits in a whole number. • Students could perform arithmetic calculations with numbers up to 3 digits. However, a few of them neglected the computational rule of doing 'multiplication before addition / subtraction'. • Students were capable of solving application problems involving mixed operations by presenting correct working steps. Some students mistook multiplication to solve application problems involving division or mixed up the minuend with the subtrahend in writing the mathematical expression. • Students understood the concept of fractions as a part of one whole and recognized the relationship between fractions and the whole. They were able to compare fractions. • Students were able to solve application problems involving the addition or the multiplication of money.	• Students were capable of recognizing the place values in whole numbers and decimals. • Students could perform arithmetic operations on whole numbers, fractions and decimals. • Some students neglected the computation rule of "performing multiplication/division before addition/subtraction" when carrying out mixed operations. • Students could understand the concept of a fraction as parts of one whole and compare fractions. • Students could solve application problems and show the working steps. • Some students were weak in presenting solutions to application problems involving fractions. • Students were capable of choosing appropriate mathematical expressions for finding an estimate.	• Students were quite good at the basic operations of directed numbers. • Students could use rate and ratio to solve simple problems. • Students could judge the reasonableness of answers from computations. • Many students were not able to estimate the values with reasonable justifications according to the question. • Students were able to use percentages to solve simple application problems. However, some students confused the formula of finding simple interest with that of compound interest.
Algebra	N.A.	• Students were capable of using symbols to represent numbers. • Students were capable of solving equations involving at most two steps in the solutions. • Students were capable of solving problems by simple equations.	• The performance of students was quite good in solving simple equations. • Students did quite well in dealing with the additions, subtractions and expansions of simple polynomials. • Students were able to substitute values into formulas and find the value of unknowns. • The performance of students was fair in recognizing the terminologies of polynomials. • Students were able to use the properties of inequalities to solve problems. Their performance in applying the laws of integral indices was fair.

Level / Dimension	P.3	P.6	S.3
Measures	• Students were capable of reading the price tags and using Hong Kong money. Some students' performance was only fair in money exchange. • Students could find the correct dates and days of a week from a calendar and tell time on a clock face and a digital clock. • Students were able to measure and compare the length and weight of objects as well as the capacity of containers. • Students were able to choose appropriate tools to measure the length and weight of objects, and the capacity of containers. • Students in general could record the length and weight of objects with appropriate units. However, some students confused the unit of length with that of weight.	• Students could write the correct dates and days of a week. • Students were able to apply the '24-hour time' and measure the time duration of activities. • Students were not capable of recording the weight and capacity of objects with appropriate units. • Students were capable of measuring and comparing the capacity of containers. • Students could apply the formula for finding the circumference. • Students could calculate the perimeter and area of simple 2-D shapes as well as the volume of cubes. • Students could apply the speed formula to solve problems.	• Students were able to choose an appropriate unit and the degree of accuracy for real-life measurements. They could select the appropriate ways to reduce errors in measurements. • Students were able to calculate arc lengths, areas of sectors, volumes of pyramids and curved surface areas of cones. • The performance of students in estimating measures with justification was only fair. • Students were weak in abstract concepts such as distinguishing among formulas for volumes by considering dimensions. • Some students could not represent a time in the '24-hour time'.

Level / Dimension	P.3	P.6	S.3
Shape and Space	• Students were able to identify pyramids/cones, prisms/cylinders and spheres. Some of them could not classify prisms, pyramids, cylinders and cones. • Some students confused 3-D shapes with 2-D shapes. • Students were capable of identifying different 2-D shapes including trapeziums and rhombuses. Some of them confused rectangles with parallelograms. • Students in general could identify straight lines and curves; parallel lines and perpendicular lines. • Students were able to recognize right angles and compare the size of angles. • Students were able to recognize the four directions, namely, north, east, south and west.	• Students could recognize cones, pyramids, cylinders, prisms and spheres. • Students were capable of identifying 2-D shapes and recognizing their characteristics. • Students could recognize the eight compass points.	• Students could not demonstrate recognition of some common terms in geometry such as regular polygons, concave polygons, straight angles etc. • Students could identify the relation between simple 3-D solids and their corresponding 2-D figures. • Students could sketch simple solids. • Students could deal with simple symmetry and transformation. • Some students confused the reasons for congruent triangles with that for similar triangles. • Students had good knowledge of the rectangular coordinate system. However, their performance in finding areas of simple figures was only fair. • Students did well in solving simple geometric problems like using the angle properties associated with intersecting lines/parallel lines, using the relations between sides and angles associated with isosceles triangles. • Students could write the correct steps of a geometric proof and use the conditions for congruent triangles to perform simple proofs, but many of them still could not provide sufficient reasons or complete the proof correctly.

Level / Dimension	P.3	P.6	S.3
Data Handling	• Students were capable of reading pictograms with one-to-one representation. They could retrieve data from the pictogram to answer simple questions. • Students were able to construct pictograms using one-to-one representation by referring to the given raw data.	• Students were capable of reading pictograms and bar charts, including those of greater frequency counts. • Students could extract the information provided in statistical graphs in order to answer questions. • Students could construct pictograms and bar charts with suitable titles. • Students were capable of finding the average of a group of data and solving simple problems of averages.	• Students recognized the basic procedures of statistical work. • Quite a number of students were not able to distinguish between discrete and continuous data. • Students could interpret simple statistical charts and choose appropriate diagrams / graphs to present a set of data. • The majority of students could construct simple statistical charts. • Some students were not able to identify sources of deception in misleading graphs or in cases of misuse of averages. • Students' performance was quite good in calculating probabilities.